Stability of slopes Under Induced Earthquake with Anisotropic Cohesion Strength

by

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Abstract

In this paper, an attempt is made to evaluate the yield acceleration and the corresponding failure mechanism in a slope by the upper bound techniques of pseudo-static limit analysis in addition to the privious paper. The term "anisotropic" means anisotropic cohesion strength and the term "nonhomogeneous" means that the cohesion strength linealy varys in the direction of depth.

The method of limit analysis is derived from the assumption that soil deformation obeys the flow rule associated with the Coulomb yield condition and volume increases as a plastic shearing deformation takes place. Then, a soil mass slides as a rigid body and its motion occurs with an angle ϕ between the velocity vector and the discontinuous slip surface.

Thus, obtained optimize solution by "reduced gradient method" and/or by "modified powell's method" are in good agreement. Some results are compared with the ones calculated previously by several investigators. They are also in good agreement.

I. Introduction

During earthquakes, ground movements can induce large inertia forces in slopes. As a result, the inertia forces moving away from the slope tend to reduce the stability of the slope. Once the inertia forces exceed the limit of the soil resistance, slope failures occur.

While the limit equilibrium method has been widely used for solving soil stability problems for more than 200 years, the application of the upper bound limit analysis technique, which is originally proposed for metals, in soil mechanics is a recent one.

In this study, we are concerned with the calculation of the critical or yield horizontal inertia force corresponding to the yield acceleration factor K_c, at which a condition of incipient slope movement is possible along the potential sliding surface. The critical mode of failure depends on the properties of soil, slope angle, magnitude of inertia force, surcharge, changes of cohesion strength and heigth of slope etc.. And the failure occurance is found from the evaluation of the safety factor of slopes which is denoted by K_c ; the factor of yield acceleration during earthquakes.

In this method, the effect of an earthquake on a potential sliding mass are represented by an

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equivalent static horizontal force defined as the product of a seismic coefficient factor K, and the weight of the potential sliding mass.

Specifically, the plastic limit theorems of the limit analysis are based on the following four basic assumption.

(1) Changes in geometry of the plastic medium are negligible.
(2) The material is perfectly plastic and obeys the Mohr-Coulomb yield criterion for soils.
(3) The plastic strain rate is normal to the yield surface or, the Mohr-Coulomb failure envelope to which the stress rate is tangential.
(4) The direction of the principal strain rate are in compliance with those of the principal stress axes.

Also, in this study, no consideration is given to the effects of vertical motion induced by the earthquake.

II. Theoretical Expression

In this paper, the computation of the yield acceleration factor K_c by the upper bound technique of limit analysis for nonhomogeneous, anisotropic soils is based on the case of log-spiral failure mechanism passing below the toe (Fig. 1, a).

The stability evaluation of a slope subjected to earthquake loads is based on the following conditions :

(1) Plane strain condition
(2) Upper bound technique of limit analysis
(3) Pseudo-static earthquake loading
(4) Uniform horizontal distribution of lateral acceleration
(5) Mohr-Coulomb criterion for failure with variable c but constant ϕ.

Upper bound limit analysis solutions of earthquake-induced failures of slopes and retaining structures corresponding to a homogeneous isotropic soil are reported elsewhere[1)2)].

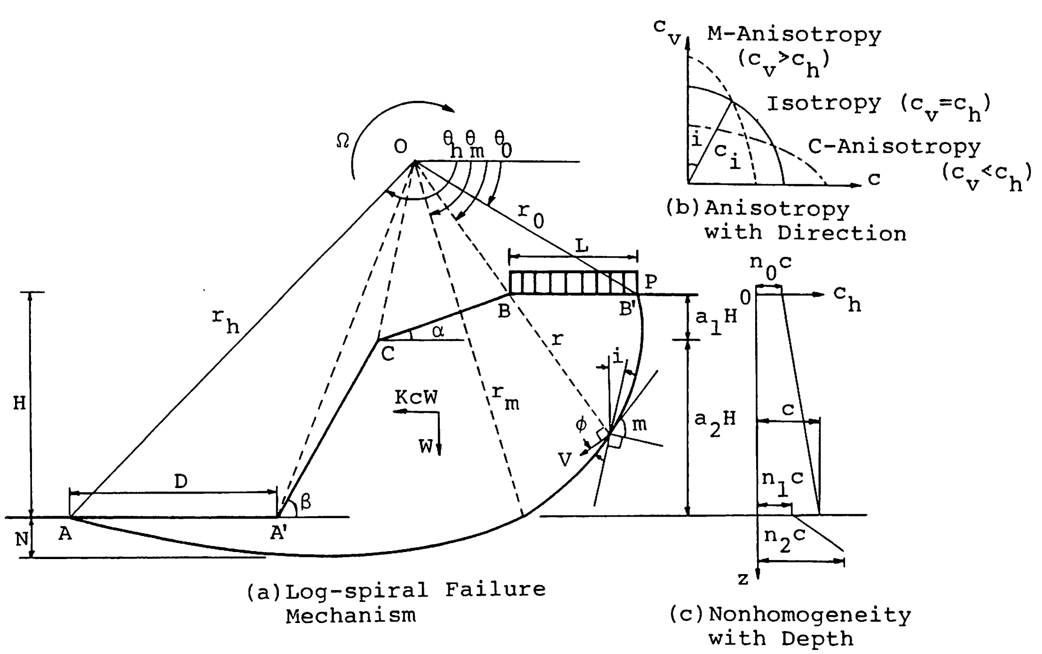

Fig. 1 : A log-spiral failure mechanism for a general slope

Herein, the term "Nonhomogeneous soil" means only the cohesion strength, c which is assumed to vary linearly with depth (Fig. 1, c). Figure 2 summarizes diagrammatically some of the simple cutting in normally consolidated clays with several forms of cohesion strength distribution being considered previously by several investigators[8)10)11)14)].

The term "anisotropic soil" implies here the variation of the cohesion strength, c, with direction at a particular point. The anisotropy with respect to cohesion strength, c, of the soil has been studied by several investigators[9)10)11)]. It is found that the variation of cohesion strength, c, with direction approximates to the curve shown in Fig. 1, b. In this paper, the variation of the apparent friction angle is not considered with respect to either the nonhomogeneity or the anisotropy. In the following we assume that the cohesion strength c_i, with its major principal stress inclined at an angle i with the vertical direction, is given by

$$c_i = c_h + (c_v - c_h) \cos^2 i \tag{1}$$

where c_h and c_v are the cohesion strength in the horizontal and vertical directions, respectively. The cohesion strengths may be termed as "principal cohesion strengths[9)]".

For example, the vertical cohesion strength, c_v can be obtained by taking vertical soil samples at any position and being investigated with the major principal stress applied in the same direction. The ratio of the principal cohesion strength c_h/c_v denoted by k, is assumed to be the same at all points in the medium. $c_i = c_h = c_v$ or k=1.0 means an isotropic material. In Fig. 1, a, the angle m is the angle between the failure plane and plane which is normal to the direction of the major principle cohesion strength kept at an angle i with the vertical direction. This angle, according to Lo's test (1965)[9)], is found to be independent of the angle of rotation of the major principal stress.

The geometrical relation L/r_o, H/r_o and N/r_o in Fig. 1, a can be shown in the following forms from (2) to (4).

The rate of external work done by the region AA'CBB' can be obtained from the algebraic summation of $\dot{W}_1$-$\dot{W}_2$-$\dot{W}_3$-$\dot{W}_4$-$\dot{W}_5$: Herein, $\dot{W}_1$, $\dot{W}_2$, $\dot{W}_3$, $\dot{W}_4$ and $\dot{W}_5$ represent the rates of external work done by the soil weight in the region OAB, OB'B, OCB, OA'C and OAA' respectively.

Similarly, the rate of external work done by force on the soil weight can be found simple summation by $\dot{W}_6$-$\dot{W}_7$-$\dot{W}_8$-$\dot{W}_9$-$\dot{W}_{10}$. Herein, $\dot{W}_6$, $\dot{W}_7$, $\dot{W}_8$, $\dot{W}_9$ and $\dot{W}_{10}$ represent the rates of external work done by the inertia force due to sliding soil weight in regions OAB', OBB', OCB, OA'C and OAA', respectively.

These expressions are as follows from (5) to (14).

$$\frac{L}{r_o} = \cos\theta_o - \cos\theta_h \exp[(\theta_h - \theta_o)\tan\phi] - \frac{D}{r_o} - \frac{H}{r_o}(a_1 \cos\beta_1 + a_2 \cos\beta_2) \tag{2}$$

$$\frac{H}{r_o} = \sin\theta_h \exp[(\theta_h - \theta_o)\tan\phi] - \sin\theta_o \tag{3}$$

$$\frac{N}{r_o} = \cos\phi \exp\left[\left(\frac{\pi}{2} + \phi - \theta_o\right)\tan\phi\right] - \sin\theta_o - \frac{H}{r_o} \tag{4}$$

where a_1, a_2, D and N are defined in Fig. 1,a.

$$\dot{W}_1 = \gamma\Omega\, r_o^3\left[\frac{1}{3(1+9\tan^2\phi)}\{(3\tan\phi\cos\theta_h + \sin\theta_h)\exp[3(\theta_h - \theta_o)\tan\phi] - 3\tan\phi\cos\theta_o - \sin\theta_o\}\right]$$
$$= \gamma\Omega\, r_o^3 G_1 \tag{5}$$

$$\dot{W}_2=\gamma r_o^3\Omega\frac{1}{6}\sin\theta_o\frac{L}{r_o}\left\{2\cos\theta_o-\frac{L}{r_o}\right\}$$
$$=\gamma r_o^3\Omega G_2 \tag{6}$$

$$\dot{W}_3=\gamma r_o^3\Omega\left[\frac{a_1}{3}\frac{H}{r_o}\left\{\cos^2\theta_o+\frac{L}{r_o}\left(\frac{L}{r_o}-2\cos\theta_o\right)+\sin\theta_o\cot\beta_1\left(\cos\theta_o-\frac{L}{r_o}\right)\right.\right.$$
$$\left.\left.-\frac{a_1}{2}\frac{H}{r_o}\cot\beta_1\left(\cos\theta_o-\frac{L}{r_o}+\sin\theta_o\cot\beta_1\right)\right\}\right]$$
$$=\gamma r_o^3\Omega G_3 \tag{7}$$

$$\dot{W}_4=\gamma r_o^3\Omega\frac{a_2}{3}\frac{H}{r_o}\left[(\cos^2\theta_h+\cot\beta_2\sin\theta_h\cos\theta_h)\exp[2(\theta_h-\theta_o)\tan\phi]\right.$$
$$+\left(\frac{D}{r_o}\cot\beta_2\sin\theta_h+\frac{a_2}{2}\frac{H}{r_o}\sin\theta_h\cot^2\beta_2+2\frac{D}{r_o}\cos\theta_h+\frac{a_2}{2}\frac{H}{r_o}\cot\beta_2\cos\theta_h\right)$$
$$\left.\cdot\exp[(\theta_h-\theta_o)\tan\phi]+\left(\frac{D}{r_o}\right)^2+\frac{a_2}{2}\frac{H}{r_o}\cot\beta_2\left(\frac{D}{r_o}\right)\right]$$
$$=\gamma r_o^3\Omega G_4 \tag{8}$$

$$\dot{W}_5=\gamma r_o^3\Omega\frac{1}{6}\frac{D}{r_o}\sin\theta_h\left[\{2\cos\theta_h\exp[(\theta_h-\theta_o)\tan\phi]+\frac{D}{r_o}\}\exp[(\theta_h-\theta_o)\tan\phi]\right]$$
$$=\gamma r_o^3\Omega G_5 \tag{9}$$

$$\dot{W}_6=K\gamma r_o^3\Omega\left[\frac{1}{3(1+9\tan^2\phi)}\{(3\tan\phi\sin\theta_h-\cos\theta_h)\exp[3(\theta_h-\theta_o)\tan\phi]\right.$$
$$\left.-3\tan\phi\sin\theta_o+\cos\theta_o\}\right]$$
$$=K\gamma r_o^3\Omega G_6 \tag{10}$$

$$\dot{W}_7=\frac{K\gamma r_o^3\Omega}{3}\left\{\frac{L}{r_o}\sin\theta_o\sin\theta_o\right\}$$
$$=K\gamma r_o^3\Omega G_7 \tag{11}$$

$$\dot{W}_8=K\gamma r_o^3\Omega\left[\frac{a_1}{3}\frac{H}{r_o}\left\{\sin\theta_o+\frac{a_1}{2}\left(\frac{H}{r_o}\right)\right\}\left(\cos\theta_o+\sin\theta_o\cos\beta_1-\frac{L}{r_o}\right)\right]$$
$$=K\gamma r_o^3\Omega G_8 \tag{12}$$

$$\dot{W}_9=\frac{K\gamma r_o^3\Omega}{3}\left[\sin\theta_h\cos\theta_h\left(\sin\theta_o+\frac{H}{r_o}\right)\exp[2(\theta_h-\theta_o)\tan\phi]\right.$$
$$+\exp[(\theta_h-\theta_o)\tan\phi]\left[\sin\theta_h\frac{D}{r_o}\left(\sin\theta_o+\frac{H}{r_o}\right)-\cos\theta_h\left\{\left(\sin\theta_o+(a_2+1)\frac{H}{r_o}\right)\right.\right.$$
$$\left.\left.\cdot\left(\sin\theta_o+\frac{H}{r_o}\right)+\frac{1}{2}\left(\frac{a_2H}{r_o}\right)^2\right\}\right]-\frac{D}{r_o}\left\{\left(\sin\theta_o+(a_2+1)\frac{H}{r_o}\right)\left(\sin\theta_o+\frac{H}{r_o}\right)+\frac{1}{2}\left(\frac{a_2H}{r_o}\right)^2\right\}$$
$$\left.-a_2\cot\beta_2\frac{H}{r_o}\left\{\sin\theta_o+\left(1-\frac{a_2}{2}\right)\frac{H}{r_o}\right\}\left(\sin\theta_o+\frac{H}{r_o}\right)\right]$$
$$=K\gamma r_o^3\Omega G_9 \tag{13}$$

$$\dot{W}_{10}=K\gamma r_o^3\Omega\left\{\frac{1}{3}\frac{D}{r_o}\sin^2\theta_h\exp[2(\theta_h-\theta_o)\tan\phi]\right\}$$
$$=K\gamma r_o^3\Omega G_{10} \tag{14}$$

The external rate of work due to surcharge boundary loads and its associated inertia force are found to be as follows.

$$p\, r_o^2 \Omega \left\{ \frac{L}{r_o} \left(\cos\theta_o - \frac{L}{2r_o} \right) \right\} = p\, r_o^2 \Omega\, f_p \tag{15}$$

$$xKp\, r_o^2 \Omega \left\{ \frac{L}{r_o} \sin\theta_o \right\} = p\, r_o^2 \Omega\, f_q \tag{16}$$

Where xK is the yield acceleration factor corresponding to the surcharge load P whose magnitude can be related to the yield acceleration factor of the sliding soil weight, K, through the coefficient, x. The value of x may be taken any value from zero representing the inertia response of surcharge load to the earthquake force.

The total rates of internal energy dissipation along the discontinuous log-spiral failure surface AB′ is found by multiplying the differential area $rd\theta/\cos\phi$ by c_i times the discontinuity in velocity, $V\cos\phi$, across the surface and integrating over the whole surface AB′. Since the layered clays possess different values of c_i, the integration is therefore carried out into two parts.

$$\int_{\theta_o}^{\theta_h} c_i (V\cos\phi) \frac{rd\theta}{\cos\phi} = \int_{\theta_o}^{\theta_m} (c_i)_I r_o v_o \exp[2(\theta-\theta_o)\tan\phi]$$

$$+ \int_{\theta_m}^{\theta_h} (c_i)_{II} r_o v_o \exp[2(\theta-\theta_o)\tan\phi] d\theta \tag{17}$$

$(c_i)_I$ and $(c_i)_{II}$ can be expressed as (18) and (19).

$$(c_i)_I = \left\{ 1 + \left(\frac{1-k}{k} \right) \cos^2 i \right\} c \left\{ n_o + \frac{1-n_o}{\frac{H}{r_o}} (\sin\theta \exp[(\theta-\theta_o)\tan\phi] - \sin\theta_o) \right\} \tag{18}$$

$$(c_i)_{II} = \left\{ 1 + \left(\frac{1-k}{k} \right) \cos^2 i \right\} c \left\{ n_1 + \frac{n_2-n_1}{\frac{N}{r_o}} (\sin\theta \exp[(\theta-\theta_o)\tan\phi] - \sin\theta_m \exp[(\theta_m-\theta_o)\tan\phi] \right\} \tag{19}$$

where $k = \frac{c_h}{c_v}$, $i = \theta + \Phi$, $\Phi = -\left(\frac{\pi}{2} + \phi - m \right)$ and n_o, n_1 and n_2 are defined in Fig. 1,c. (20)

After integration and some simplifications, Eq. (17) reduces to as follow.

$$\int_{\theta_o}^{\theta_h} c_i (V\cos\phi) \frac{rd\theta}{\cos\phi} = c r_o^2 \Omega Q \tag{21}$$

in which

$$Q = Q_1 + Q_2 + Q_2 \tag{22}$$

The functions Q_1, Q_2 and Q_3 are shown as functions of θ_o, θ_m and K.
Also, Q_1, Q_2 and Q_3 include functions ξ, ψ, ρ and λ which are expressed as function of Q_o and Q_m, too x (see rsference, 15).

The log-spiral angle (θ_m) and the anisotropic angle (i) are related from the geometric configuration shown in Figs. l,a,b.

$$\sin\theta_m \exp[\theta_m \tan\phi] = \sin\theta_h \exp[\theta_h \tan\phi] \tag{23}$$

By equating the total rates of external work, Eq. (5) to (16) to the total rate of internal energy dissipation, Eq. (21), we obtain.

$$K = F\left(\theta_o,\ \theta_h,\ \frac{D}{r_o} \right) = \frac{c(Q_1+Q_2+Q_3) - \gamma r_o (G_1 - G_2 - G_3 - G_4 - G_5) - p f_p}{\gamma r_o (G_6 - G_7 - G_8 - G_9 - G_{10}) + x p f_q} \tag{24}$$

The function $F\left(\theta_o, \theta_h, \frac{D}{r_o}\right)$ has a minimum value and, thus, indicates a least upper bound, when θ_o, θ_h, and $\frac{D}{r_o}$ satisfy the following conditions.

$$\frac{\partial F}{\partial \theta_o}=0\ ;\ \frac{\partial F}{\partial \theta_h}=0 \text{ and } \frac{\partial F}{\partial D/r_o}=0 \tag{25}$$

Thus, the yield acceleration factor, K_c is denoted as

$$K_c=\text{Min. } F(\theta_o,\ \theta_h,\ D/r_o) \tag{26}$$

III. Numerical Results and Summary

Computer programs were developed at Purdue University, Hokkaido University and Muroran Institute of Technology. The program includes three parts (1) a main program, (2) a function subprogram which defines the objective function and can calculate the minimum acceleration factor, and (3) a subroutine subprogram which decides the constraints. The main program serves two purposes, (1) initialization of program parameters and (2) preparation of calling subroutine OPT, OPTM and SUMT which are packages of subroutine performing the generalized reduced gradient method and/or modified gradient method for the solution of a constraint and/or a unconstraint nonlinear programming problem. To submit a problem, the user only needs to supply the input data defining soil properties and geometrical relations of slope. The objective function also needs constrains which have already been furnished details of the program including its listings are given

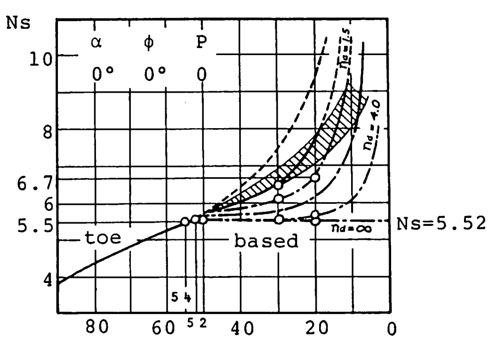

Fig. 2 : Comparison with existing analysis (D. W. Taylor, 1948) (o : values of the present limit analysis)

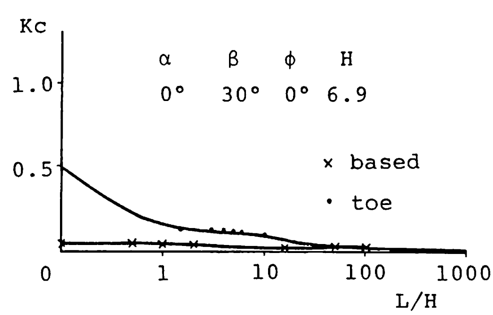

Fig. 3 : Relationship between K_c and L/H

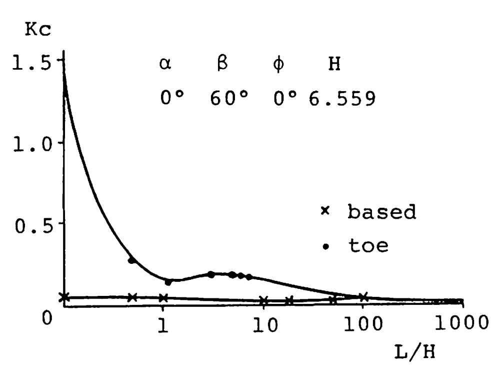

Fig. 4 : Relationship between K_c and L/H

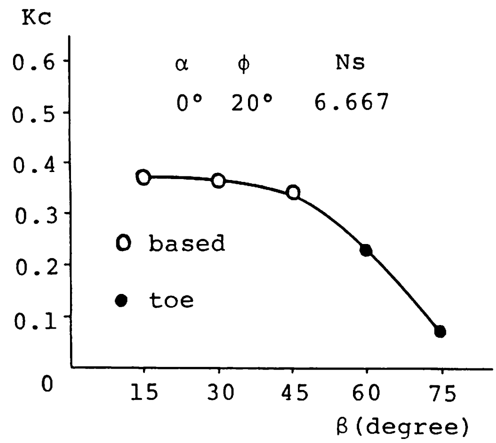

Fig. 5 : Relationship between K_c and β

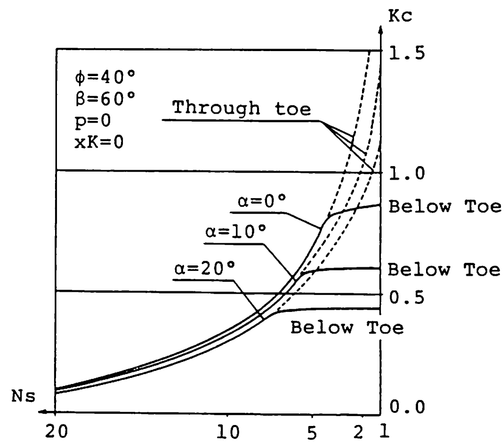

Fig. 6 : Relationship between K_c and Ns

elsewhere[5].

Extensive numerical results have been obtained by this program. The results are summarized in Fig. 2 to 6 and Table 1 to 2. Some of the solutions are compared in Table 1 and Table 2 with the existing limit equilibrium solutions.

(1) The relation between the stability number Ns and the slope angle β for the limit equilibrium method[14] are compared with the one for the present limit analysis in case of $\phi=0$, $\alpha=0$, $P=0$ and the cohesion c=constant as shown in Fig. 2. Both relations quite agree to one another.

(2) The relationship between K_c and L/H is shown in Fig. 3, 4. Generally speaking, it would be sufficient to take 10 as L/H for a constraint in calculating the optimized solutions.

(3) Fig. 5 shows that the value of K_c decreases as β increases. This figure also illustrates that the failure mechanism passing below the toe (based-failure) changes to the failure mechanism passing through the toe (toe-failure), around $\beta=50^\circ$.

(4) Based on the results for these case of $k=1.0$, the variation of the stability number N_s with yield acceleration K_c is shown in Fig. 6 where the solutions corresponding to the toe-failure are cut off by the solutions based-failure for different upper slope angle α.

(5) Table 1 shows a comparison of critical heights obtained by the limit equilibrium method with by the present limit analysis for the slope of anisotropic and homogeneous cohesion, in which the former one performed by LO[9]. Going into detail, as in LO's work, the value of m (see Fig. 1, a) is taken to be 55° and the value of friction angle ϕ and acceleration K_c are put nearly equal to zero so that the statical log-spiral failure surface reduces to the circular one. Generally speaking, both results are in good agreement.

(6) Table 2 gives the comparison of the results for the slope of anisotropic cohesion which increases linearly with depth between two method of evaluation as in (5). Another word, the critical height H_c is compared with the results according to LO (1965) by means of the limit equilibrium method. A good agreement is again observed.

Table 1 Comparson of critical height : H_c for anisotropic soil with constant shear strength.

Slope angle (degree) β	Anisotropy factor k	Curved failure surface: Limit :1 equilibrium ϕ circle	Limit :2 analysis log spiral	Ratio of 1 / 2
90	1.0	95.75	110.75	0.870
	0.9	--	--	--
	0.8	--	--	--
	0.7	--	--	--
	0.6	--	--	--
	0.5	--	--	--
70	1.0	119.75	136.62	0.877
	0.9	113.00	132.36	0.892
	0.8	116.25	123.14	0.907
	0.7	114.50	123.89	0.924
	0.6	112.25	119.12	0.942
	0.5	110.25	114.92	0.960
50	1.0	142.00	142.00	1.000
	0.9	138.50	137.50	1.007
	0.8	133.75	129.40	1.034
	0.7	129.75	125.50	1.054
	0.6	129.75	125.50	1.054
	0.6	127.25	120.75	1.054
	0.5	121.25	116.50	1.041

*Lo (1965).

Table 2 Comparision of critical height : H_c for anisotropic soil with shear strength increasing linearly with depth.

Slope angle (degree) β	Anistropy factory k	Curved failure surface: Limit :1 equilibrium ϕ circle	Limit :2 analysis log spiral	Ratio of Ratio of 1 / 2
90	1.0	50.00	60.97	0.820
	0.9	50.00	60.45	0.827
	0.8	50.00	60.30	0.829
	0.7	50.00	59.40	0.842
	0.6	50.00	58.85	0.850
	0.5	50.00	58.35	0.857
70	1.0	69.25	72.10	0.961
	0.9	68.25	72.06	0.947
	0.8	67.25	70.77	0.950
	0.7	66.25	70.40	0.941
	0.6	65.25	70.20	0.930
	0.5	62.25	68.68	0.910
50	1.0	94.50	103.70	0.911
	0.9	91.50	100.50	0.911
	0.8	89.00	98.00	0.908
	0.7	86.25	95.40	0.904
	0.6	82.75	92.40	0.896
	0.5	79.25	89.50	0.866
30	1.0	137.50	135.50	1.015
	0.9	--	--	--
	0.8	125.00	127.00	0.984
	0.7	--	--	--
	0.6	--	--	--
	0.5	104.50	114.00	0.917

*Lo (1965).

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