Assesment for Seismic Bearing Capacity of a Foundation with a Slope

by

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Abstract

The finite element technique may be mighty way to find deformation of the element in the structure and evaluate its safety through the computation[10]. It may, however, be for more difficult and complicated to find a progressive failure concerning soil structures. The conventional pseudo-static approach[4,5] is indeed a simple way to compute the upper limit of a slope under seismic load but may not informs of a sliding displacement as such. In this paper, an upperbound technique of the limit analysis[3,8] is used to find bearing capacity of a foundation near a down-hill slope is discussed upon the assumptions that a logarithmic spiral rapture starts at an edge of the loaded area far from the slope, a sliding part of soil behaves as a rigid body and inertia force acts at its center of gravity. The expression that the rate of kinematic energy of the sliding soil block together with the load should be equal to the dissipation of internal energy rate along the sliding line, leads to the equation of ultimate load by optimizing the various parameters in it. Thus obtained numerical results are compared with those by Kötter's method as well as a kind of finite element method.

1. INTRODUION

For the method of theoretically estimating the limit bearing force of a foundation on a slope, the finite element analysis and boundary element analysis with elastoplastic elements or rigid body-spring elements have been widely used[1,2,9,10]. These methods can give the design guideline with good accuracy, but for the analysis, fairly troublesome calculation is required, and it is unavoidable that the amount of arithmetic becomes large. Besides, there are also the problems of respective accuracies, and as to which phase is taken as the upper boundary, especially as to the determination of a safety factor at the time of earthquakes, it seems that a certain criterion has not yet been obtained. In the present draft plan, the bearing capacity is calculated by the equation for the bearing capacity coefficient in the formula for bearing capacity according to the specifications for road bridges, multiplied by the correction factor showing the effect of a slope.

In this paper, the limit load Pc when a load acts on the foudation on a slope as shown in Figure 1 is expressed as the solution of a function having θ_0 and θ_h showing the form of the slope as paramenters by using the upper boundary method of limit analysis[4,6], and this smallest upper boundary value is to be determined as a nonlinear optimization problem taking the margin width b in consideration, which is a part on which the foundation is not placed on the levee crown of the slope.

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Moreover, this analitical method can determine similarly the limit load also when a horizontal earthquake acceleration is considered for a mound foundation and load system at the time of earthquakes. In this case, it is assumed that the sliding earth mass is a perfect plastic body, in conformity with the breaking criteria of Mohr-Coulomb, and coincider with the principal axis direction of a plastic strain increment, beside, the geometrical deformation of a plastic medium is neglected.

According to the above description, the bearing capacity coefficient at the limit bearing capacity of a slope was determined, and the examination of this analitical metod assuming that a collapsing sliding surface is a logarithmic special is carried out by the comparison with the conventional dividing method(7).

2. ANALYTICAL THEORY

When the own weight of the sliding region ABC shown in Figure 1 and the the power due to earthquake inertia force about the turning center 0 are denoted by W and W′, respectively, W and W′ are obtained by subtracting the power for the region OBC and that for the region OAB from that about the point 0 which is calculated by assuming that the whole region OAC is composed of earth, and are expressed by the following forms.

$$\dot{W} = \dot{W}_{OAC} - \dot{W}_{OBC} - \dot{W}_{OAB} = \gamma r_0^3 \Omega (f_1 - f_2 - f_3) \tag{1}$$

$$\dot{W}' = \dot{W}'_{OAC} - \dot{W}'_{OBC} - \dot{W}'_{OAB} = \gamma r_0^3 \Omega (f_4 - f_5 - f_6) \tag{2}$$

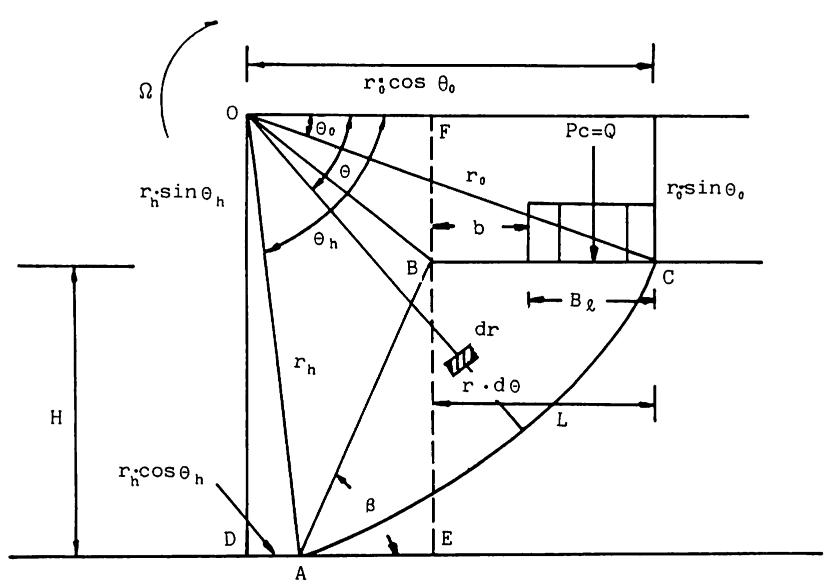


Fig. 1 General illustration

Similarly, when the power due to the load P on a foundation and that due to its inertia forec xKP are denoted by W_P and W'_P, those are shown as follows.

$$\dot{W}_P = Pr_0^2\Omega\frac{L}{r_0}(\cos\theta_0 - \frac{L}{2r_0}) = Pr_0^2\Omega f_P \quad (3)$$

$$\dot{W}'_P = xKPr_0^2\Omega\frac{L}{r}\sin\theta = xKPr_0^2\Omega f_q \quad (4)$$

Next, the internall dissipating energy E ; which is the total sum of the cohesive resistance arising along a logarithmic spiral sliding surface can be derermined as follows.

$$\dot{E}_1 = \int_s (cV_s)ds = cV_O r\int_{\theta_0}^{\theta_h}\mathrm{Exp}\left[2(\theta-\theta_0)\tan\phi\right]d\theta$$

$$= \frac{cr_0^2\cdot(V_0/r_0)}{2\tan\phi}\left\{\mathrm{Exp}\left[2(\theta-\theta_0)\tan\phi\right]-1\right\} = cr_0^2\cdot\Omega f_c \quad (5)$$

Here, γ : weight per unit volume of earth (kg/m³)

C : cohesion force of earth (Pa)

K : horizontal acceleration coefficient of earthquake (horizontal magnitude)

Ω : V_0/r_0 : angular velocity of sliding earth mass around point 0 (rad/s)

V_S, V_0 : sliding velocity and initial velocity of sliding earth mass on sliding surface (m/s)

X : mass ratio of placed load P to ground

Besides,

$$f_1 = \frac{1}{3(1+9\tan^2\phi)}\{\mathrm{Exp}[3(\theta_h-\theta_0)\tan\phi](3\tan\phi\cos\theta_h+\sin\theta_h)-3\tan\phi\cos\theta_0-\sin\theta_0\}$$

$$f_2 = \frac{L}{6r_0}(2\cos\theta_0 - \frac{L}{r_0})\sin\theta_0 \qquad f_5 = \frac{1}{6}\{2\frac{L}{r_0}\sin\theta_0\sin\theta_0\}$$

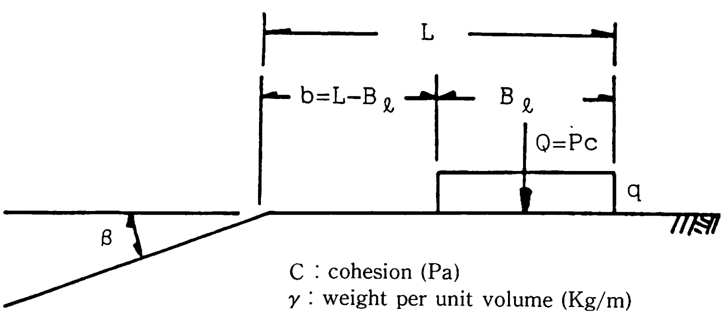
$$f_3 = \frac{1}{6}\mathrm{Exp}[(\theta_h-\theta_0)\tan\phi]\{\sin(\theta_h-\theta_0)-\frac{L}{r_0}\sin\theta_h\}\{\cos\theta_0-\frac{L}{r_0}+\cos\theta_h\,\mathrm{Exp}[(\theta_h-\theta_0)\tan\phi]\}$$

$$f_4 = \frac{1}{3(1+9\tan^2\phi)}\{(3\tan\phi\sin\theta_h-\cos\theta_h)\mathrm{Exp}[3(\theta_h-\theta_0)\tan\phi]-3\tan\phi\sin\theta_0+\cos\theta_0\}$$

$$f_6 = \frac{1}{6}\mathrm{Exp}[(\theta_h-\theta_0)\tan\phi]\{\sin(\theta_h-\theta_0)-\frac{L}{r_0}\sin\theta_h\}\{\mathrm{Exp}[(\theta_h-\theta_0)\tan\phi]\sin\theta_h+\sin\theta_0\}$$

Therefore, by equalizing the total sum of the power of sliding earth mass due to external force and the internally dissipating energy in a sliding surface, the equation of balance is determined as follows.

$$\dot{E}_1 = \dot{W} + \dot{W}_P + \dot{W}' + \dot{W}'_P \quad (6)$$



C : cohesion (Pa)

γ : weight per unit volume (Kg/m)

ϕ : internal friction angle

Fig. 2 Statical case

2－1. Solution of function in statical case

As shown in Figure 2, when the difference of the collapsing length L at the levee crown of a slope and the length of placed load distribution B_l is taken as margin width b, the case that the horizontal acceleration coefficient of earthquake K=0 is taken in the above described equation of limit balance, Equation (6), emerges. Accordingly, in the nonlinear optimization problem having a placed load P as the objective function, the solution of the function in this case is expressed as follows.

$$P(\theta_0, \theta_h) = \frac{c \cdot f_c - \gamma r_0 (f_1 - f_2 - f_3)}{f_p} \qquad (7)$$

2－2. Solution of function in case of taking inertia force at the time of earthquakes into account

Here, K_C is defined as the yield acceleration coefficient of earthquake, that is, the smallest acceleration with which a slope begins to move due to the horizontal inertia force of the earthquake divided by the acceleration of gravity. Then, the objective function P is shown as the following equation, taking an inertia term into account by the similar way of thinking from Figure 3.

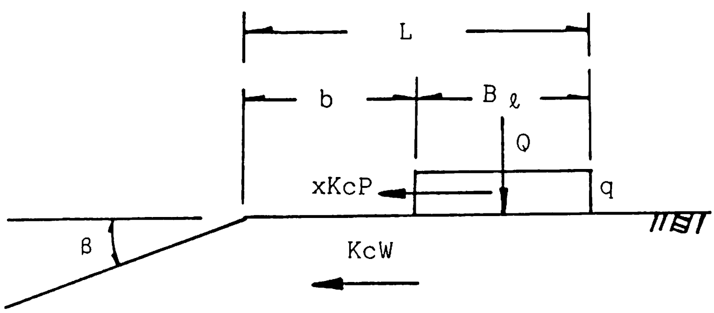


Fig. 3 Case of taking earthquake inertia force into account

$$P(\theta_0, \theta_h) = \frac{c \cdot f_c - \gamma (f_1 - f_2 - f_3) - K_c \gamma (f_4 - f_5 - f_6)}{f_p + x K_c f_q} \qquad (8)$$

Therefore, when the following conditions are satisfied, P (θ_0, θ_h) takes the minimum value.

$$\frac{\partial P}{\partial \theta_0} = 0 \qquad \frac{\partial P}{\partial \theta_h} \qquad (9)$$

Accordingly, the limit value load P_c, that is, the bearing capacity Q is expressed as follows.

$$P_c = Q = \min . P(\theta_0, \theta_h) \qquad (10)$$

On the other hand, L/r_0 and the depth H_h of a logarithmic spiral sliding surface are shown as the following equations.

$$\frac{L}{r_0} = \frac{\sin(\theta_h - \theta_0)}{\sin \theta_h} - \frac{\sin(\theta_h + \beta)}{\sin \theta_h \sin \beta} \{-\sin \theta_0 + \sin \theta_h \, \mathrm{Exp}[(\theta_h - \theta_0) \tan \phi]\} \qquad (11)$$

$$H_h = r_0 \{\mathrm{Exp}[\frac{\pi}{2} + \phi - \theta_0] \sin(\frac{\pi}{2} + \phi) - \sin \theta_0\} \qquad (12)$$

3. RESULTS OF ANALYSIS AND EXAMINATION

Hereinafter, the results of analysis are shown. The computing program for the numerical calculation is composed of three main parts as follows : (1) prime program, (2) the subprogram for determining an objective function and calculating the smallest value load and (3) the subprogram which presents restricting conditions. Here, the prime program has two objectives : (a) the determination of the initial values of parameters and (b) the analysis by B・F・G・S technique of quasi-Newtonian method. In short, the solution set of a load when the safety factor becomes 1.0 at the limit stationary state of a slope is determined, uniaxial searhc is carried out on a secondary plane, and the limit load P_c is to be determined by third order approximation. This is the upper boundary method of limit analysis, and the smallest value in all the values satisfying the conditions is to be adopted.

3 − 1. Comparison of the results of statical analysis with those of conventional method

In oredr to compare with the results of presenting analysis with those by conventional division method recommended by the Honshi Connecting Bridge Investigation and Research Subcommittee (hereinafter, referred to as Honshi), by the calculation was carried out which submerged slope with given $\gamma = 0.0$. In Figure 4, the calculated results for the various particular cases are shown, and the comparison of the analytical results is shown in Table 1 and Figure 5. In the simplified method of Honshi, the analysis was carried out, assuming that all the statically indeterminate forces in the divided blocks were mutually balanced, and the absolute values largely differed from the results by Bishop's method taking horizontal statically indeterminate forces into account, but the correction factor α being used for the actual design calculation showed the approximate value.

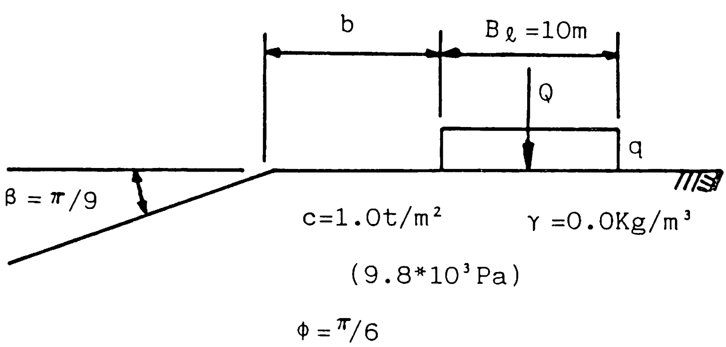


Fig. 4 Various particulars for calculation

The present analytical values are close to the values of Bishop and Kötter, and α from the presenting analytical values well coincided with the one from other analytical values.

Figure 6 shows the comparison of the sliding line in this analysis with that by division method. It can be said that on the whole the shapes of α are relatively quite similar, and as to the deepest distance in the sliding form, 19.74 m of the present analytical value assuming a loga-rithmic spiral sliding nearly coincided with 20.65 m of the results of method of Honshi assuming the compound sliding surface of circular arc and straight line.

In Table 2, the results of analysis with the variation of internal friction angle ϕ are tabulated. In this table, it is seen that the influence of ϕ becomes large. Besides, the various particulars in the numerical calculation are given as shown in Figure 7. It is shown that as ϕ becomes larger, the volume of sliding earth mass increases, that is accompanying with the increment of ϕ, the bearing capacity Q takes the larger value and the stability of a slope increases (see Figure 8).

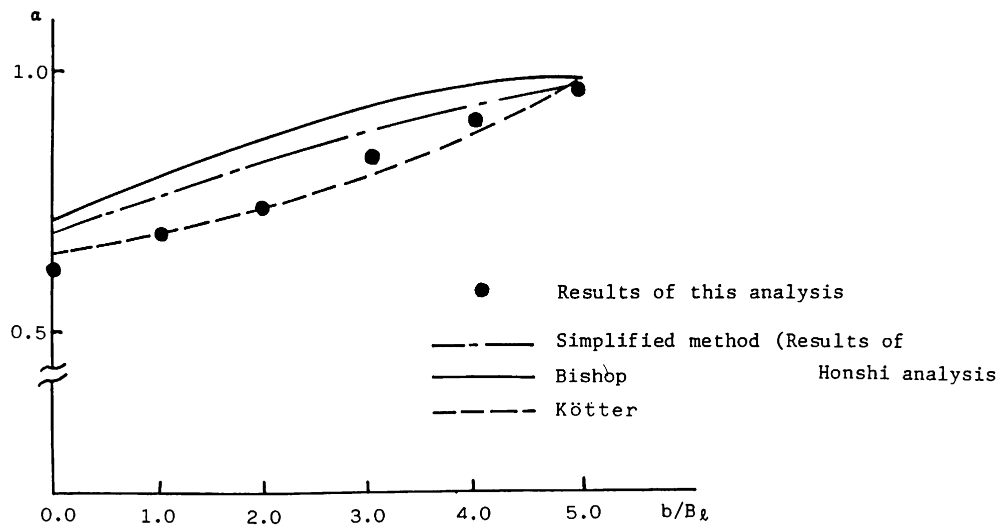


Fig. 5 Comparison of this analysis values and conventional methods (division method)

Table 1 Comparison of values of this analysis and conventional methods (division method)

Solution method \ b/Bℓ		Unifinite horizontal earthquake (∞)	0.0	1.0	2.0	5.0
Simplified method	Q t/m² (Pa)*10³	17.9 (175.5)	12.6 (123.6)	13.7 (134.4)	14.7 (144.2)	17.7 (173.6)
	Ratio a	1.00	0.70	0.77	0.82	0.99
Bishop	Q t/m² (Pa)*10³	30.3 (297.1)	21.8 (213.8)	24.5 (240.3)	26.7 (259.9)	30.3 (297.1)
	Ratio a	1.00	0.72	0.81	0.88	1.00
Kötter	Q t/m² (Pa)*10³	30.3 (297.1)	19.6 (192.2)	20.9 (205.0)	23.0 (225.6)	30.3 (297.1)
	Ratio a	1.00	0.65	0.69	0.76	1.00
Values of this analysis	Q t/m² (Pa)*10³	37.6 (368.7)	23.0 (225.6)	25.9 (254.0)	28.5 (279.5)	36.9 (361.9)
	Ratio a	1.00	0.61	0.69	0.76	0.96

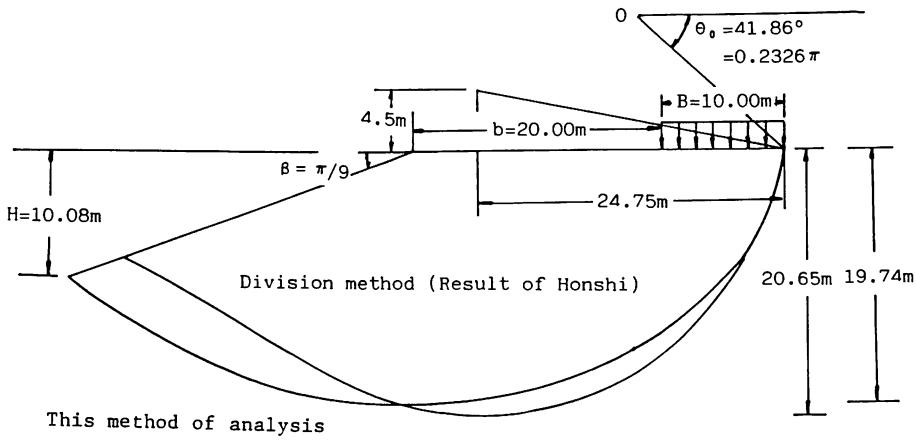


Fig. 6 Comparison of sliding lines of this analysis method and division method

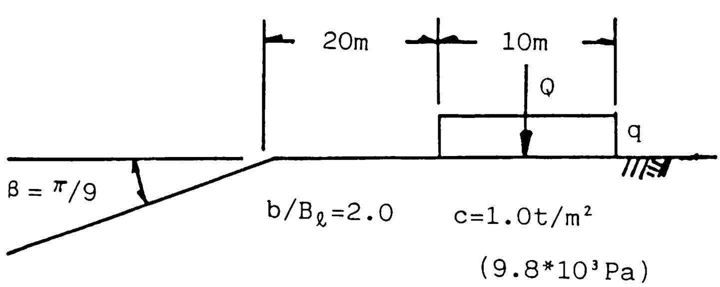


Fig. 7 Various particulars for calculation

Table 2 Change of bearing capacity bue to internal friction angle ϕ

ϕ	$\pi/36$	$\pi/18$	$\pi/9$	$\pi/6$	$2\pi/9$
Q (Bearing capacity) t/m^2 (Pa) $*10^3$	7.53 (73.84)	9.42 (92.38)	15.59 (152.89)	28.53 (279.78)	61.53 (603.40)
H_h m (Depth of slip line)	9.86	10.46	13.90	19.74	30.81

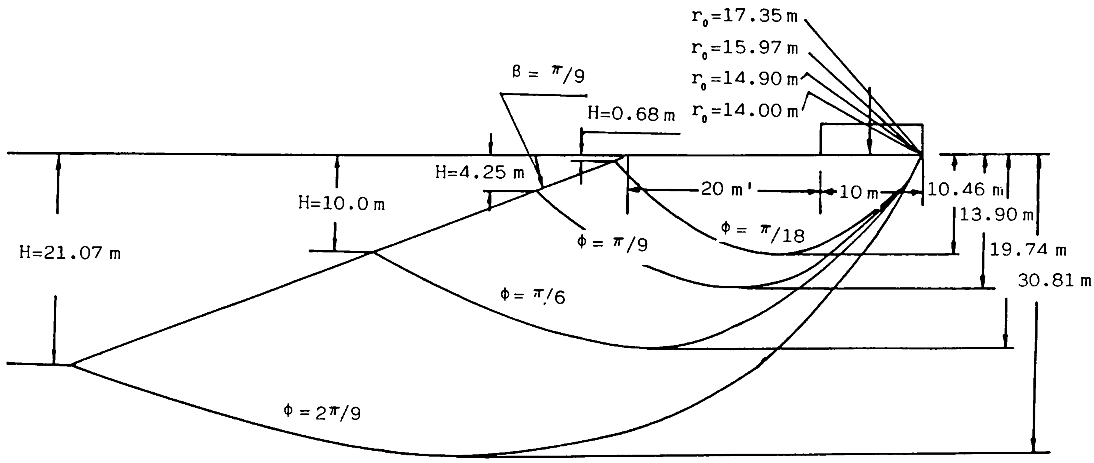


Fig. 8 Comparison of sliding due to change of internal friction angle ϕ

3 − 2. Results of analysis when inertia force due to an earthquake is taken into account

In Table 3, the comparison of the statical values without any horizontal inertia force due to an earthquake into account (k= 0) and the case of taking it into account is shown in relation with the variation of b/B_l. In Tacle 4, the analytical values in relation with the variation of horizontal inertia force due to earthquakes at $b/B_l=2.0$ are shown. As clearly seen in Tables 3 and 4, the influence of the inertia forces of sliding earth mass and a placed load is large, and it is shown that they decrease the bearing capacity. Besides, the calculation for the particulars are taken the values in Figure 9.

Moreover, in Tables 5 and 6, the results of analysis are shown with the variation of the limit earthquake acceleration coefficient the k_c and b/B_l. In both cases, the larger K_c or b/B_l becomes, the smaller the bearing capacity Q (the limit placed load P_c) becomes.

In Table 7, the analytical values in relation to the change of x, which is the mass ratio of a placed load and ground, are shown. As clearly seen in the table, accompanying the increase of x, the bearing capacity became smaller, and a slope tended to be unstable. Accordingly, it seems that also the influence of x in large.

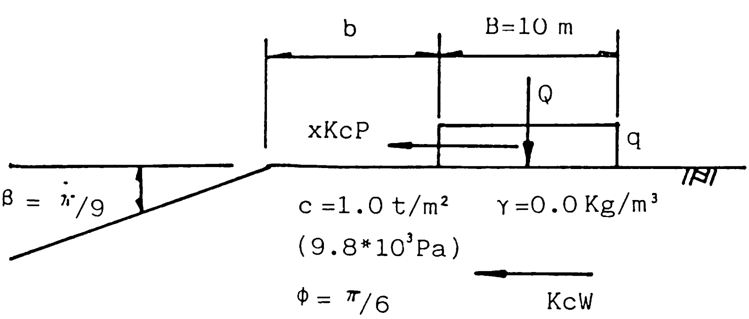


Fig. 9 Various particulars for calculation

Table 3 Bearing capacity due to change of b/B_l in case of taking inertia force into account

b/Bℓ	0.0	1.0	2.0	3.0	4.0	5.0
k=0.0 : Q t/m² (Pa) *10³	23.00 (225.55)	25.85 (253.50)	28.53 (279.78)	31.10 (304.99)	33.77 (331.17)	36.88 (361.67)
α	0.61	0.69	0.76	0.83	0.90	0.98
k=0.1 : Q t/m² (Pa) *10³	18.29 (179.36)	20.79 (203.88)	23.14 (226.93)	25.42 (249.29)	27.67 (271.35)	30.22 (296.36)
α	0.49	0.55	0.62	0.68	0.74	0.80
k=0.3 : Q t/m² (Pa) *10³	11.20 (109.84)	13.14 (128.86)	15.00 (147.10)	16.74 (164.16)	18.51 (181.52)	20.33 (199.37)
α	0.30.	0.35	0.40	0.45	0.49	0.54

Table 4 Bearing capacity due to change of limit earthquake acceleration coefficient K_c

x =1.0 b/B_l=2.0

Kc / Bearing capacity	0.1	0.15	0.20	0.25	0.35	0.40	0.45
Q t/m² (Pa) *10³	23.14 (226.93)	20.77 (203.68)	18.62 (182.60)	16.69 (163.67)	13.42 (131.61)	12.06 (118.27)	10.87 (106.60)

Table 5 Bearing capacity due to change of K_c

x=1.0 b/Bℓ = 0.0	Q t/m² (Pa)*10³
Kc = 0.1	18.99 (186.23)
Kc = 0.3	12.15 (119.15)
Kc = 0.4	9.51 (93.26)
Kc = 0.5	7.61 (74.63)

Table 6 Bearing capacity due to change of b/B_l

x=0.0 Kc = 0.5	Q t/m² (Pa)*10³
b/Bℓ = 0.0	7.61 (74.63)
b/Bℓ = 1.0	9.52 (93.36)
b/Bℓ = 2.0	11.30 (110.82)
b/Bℓ = 3.0	12.96 (127.09)
b/Bℓ = 4.0	14.53 (142.49)
b/Bℓ = 5.0	16.21 (158.97)

Table 7 Bearing capacity due to change of x

Kc	x	Q $t/m^2(Pa)*10^3$
0.1	0.1	28.09 (275.47)
	0.5	26.34 (258.31)
0.2	0.1	27.65 (271.15)
	0.5	24.21 (237.42)
	1.0	20.22 (198.29)
0.5	0.1	26.34 (258.31)
	0.3	22.16 (217.32)
	0.5	18.40 (180.44)
	1.0	11.30 (110.82)

4. SUMMARY

The above results can be summarized as follows.

(1) When a sliding line is assumed to be a logarithmic spimal sliding curve, and the analysis of the bearing capacity of a foundation on a slope is carried out by the upper bound method of limit analysis, the result approximates to the analytical result by the compound sliding surface of circular arc and straight line by division method is able to be obtained.

(2) When the correction factor taking the effect of a slope on the bearing capacity coefficient in the specifications for road bridges in consideration is determined by this method of analysis, the value relatively well coincides with the result of conventional division method and the result of Honshi draft plan is obtained.

(3) When the bearing capacity of a foundation on a slope is determined by this method of analysis, the somewhat larger value than the result of division method and others is obtained. Accordingly, the conventional method seem to be on the safe side.

(4) When the bearing capacity is determined by this method of analysis taking earthquake inertia force into account, the effect of the inertia term appeared large.

Accordingly, by taking a placed load P as the objective function, the evaluation of the bearing capacity of a foundation on a slope can be carried out simply and conveniently, moreover, also in the case of taking earthquake inertia force into account, the analytical values can be determined simply by this method of analysis.

Besides, the evaluation of the bearing capacity of a foundation installed on a slope where cohesion force is uneven and anisotropic and a multi-layer slope is considered to be the subject for the future.

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Appendix-1 NOTATIONS

The following symbols are used in this paper;
g : Acceleration of gravity.
γ: Gravity of soil per unit volume.
r_0 : Radius of rotational failure mechanism. (in Fig. 1)
ϕ : Internal friction angle.
θ_0 : Angle of starting point of failure mechanism. (in Fig. 1)
θh : Angle of ending point of failure mechanism. (in Fig. 1)
c : Cohesion strength.
l : Arm length of failure mechanism.
K : Acceleration factor of earthquake.
xK : Acceleration factor corresponding to P relating to K of soil weight multiplied by coefficient x, which can be greater or less than unit.
Ω : Angular velocity relative to the materials below the failure surface about the center of rotation 0. (in Fig. 1)
L : Length of failure mechanism.
Bl : Length on surcharge load P.
b : Length from the crown to the end of surcharge load. (in Fig. 1)
Pc : critical surcharge load (equal to Q : bearing capacity).

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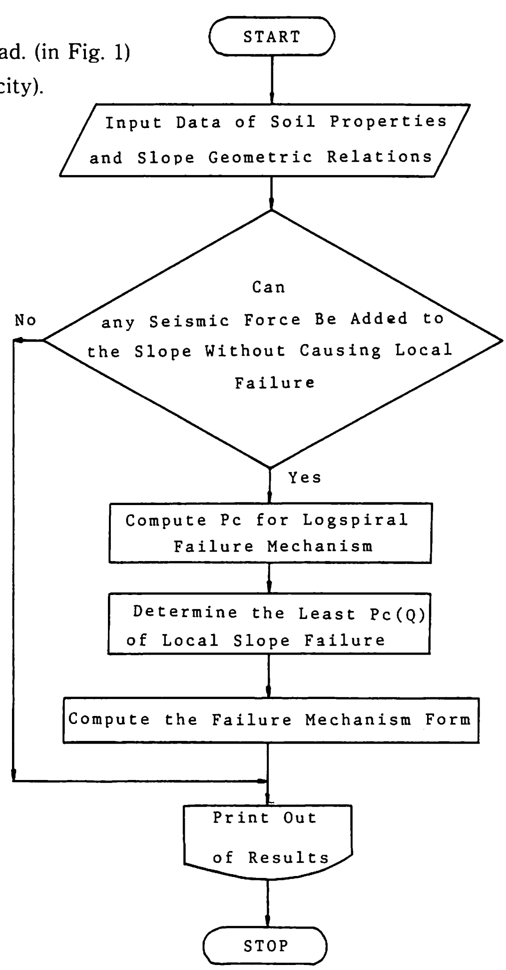

Appendix-III
Flow Chart of Computer Progrom

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