Sensitivity Analysis of Road Network Capacity by O-D Cut Matrix and Its Application to Some Traffic Management Problems

by

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Abstract

The road network has a capacity which is defined as the maximum O-D traffic volume which is able to accept the traffic demand in a given area. If a certain capacity is determined for a given road network, a traffic demand over this capacity will not be accepted. In this case, improvements in traffic flow are necessary to reduce congestion and to increase the capacity of the road network. In planning a traffic management system, the basic question to be considered is what effects the proposed improvements would have on the capacity of the road network. A sensitivity analysis is one of the approaches to this type of problem. In this paper, a sensitivity analysis of road network capacity is carried out by analyzing the effects of changes in the O-D traffic pattern on capacity ; the analysis is made on the basis of the O-D cut matrix which is proposed in this paper.

1. INTRODUCTION

A road network has a capacity for a traffic flow as an upper bound of the ability of traffic acceptance, because each node and link in the road network has a traffic capacity. The road network capacity[1),2),3)] is essential in road network planning and in transportation planning and is defined as the maximum O-D traffic volume which can be accomodated in a given area under the fixed O-D taffic pattern. Therefore, when traffic demand exceeds the capacity of the existing road network, it is necessary to make some improvements to reduce traffic jam and congestion.

Since road network capacity is defined by network characteristics and flow characteristics[3),4)], it is possible to make a balance between traffic demand and road network capacity by means of various improvements based on these two kinds of characteristics. The improvements based on network characteristics are those of widening links, constructing new links, signal coordination or introducing a one-way street system. On the other hand, the improvements based on the flow characteristics are those of parking restriction, staggered working hours, diverting the travel mode to other means, or dispersing the traffic demand, all of which are the alternatives of transportation system management (TSM).[4),5)]

In this study, arguments are presented from the latter viewpoint : flow characteristics. Various TSM alternatives which can be accomplished on a short time basis are those controlling traffic demands which exceed road network capacity ; i, e., methods to encourage changes of O-D traffic pattern by restricting trip generation or trip distribution. First, we shall discuss method of formulat-

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ing the O-D cut matrix required to quantitatively analyze the effects that changes of O-D traffic pattern have on the road network capacity. A sensitivity analysis is performed using simple matrix algebra based on the O-D cut matrix. Then, various TSM alternatives are classified and placed into either of two categories according to which O-D pair is restricted: one case restricts the specific O-D pair that diverts travel mode to other means and the other restricts the O-D pairs which are generated in and attracted to the identical zone (area) by parking restriction. The analysis for each case are made on the basis of the proposed O-D cut matrix.

Furthermore, another application attempted is the assessment of the effects on road network capacity of the relocation of land-use activities by transferring urban facilities to another zone. In this analysis, transferring between two zonal areas only is discussed. The inter-zonal sensitivity matrix for each cut is formed from the O-D cut matrix; it has been confirmed that an analysis is easily carried out using the formed inter-zonal sensitivity matrix.

2. FORMULATION OF O-D CUT MATRIX

2.1 Search for cuts by the optimal network design problem

The network design problem is concerned with the modification of transportation infrastructure by adding new links or improving existing ones. Approaches to formulating the optimal network design problem may be classified in several ways[6]. In this paper, the purpose of formulating the optimal network design problem is to search the cuts with acceptable flow greater than minimum cut (the cut that defined the road network capacity). The optimal network design problem is described as follows:

$$\text{minimize} \quad \sum_{i=1}^{m} d_i x_i \tag{1}$$

subject to

$$\sum_{r=1}^{n_k} Y_r^k = p_k \cdot G \quad (k=1,2,\cdots,q) \tag{2}$$

$$\sum_{k=1}^{q} \sum_{r=1}^{n_k} {}_i\delta_r^k Y_r^k = C_i + C_i x_i (i=1,2,\cdots,m) \tag{3}$$

$$Y_r^k \geqq 0 \quad \begin{Bmatrix} k=1,2,\cdots,q \\ r=1,2,\cdots,n_k \end{Bmatrix} \tag{4}$$

$$x_i \geqq 0 \quad (i=1,2,\cdots,m) \tag{5}$$

where

m = number of existing links on the network

d_i = length of link i

x_i = improvement made to link i (meters of road widening)

n_k = number of independent paths of O-D pair k

y_r^k = traffic volumes of O-D pair k on path r

p_k = unit O-D traffic volumes of O-D pair k

q = number of O-D pairs on the network

$${}_i\delta_r^k = \begin{cases} 1\text{; if link i is on path r of O-D pair k} \\ 0\text{; otherwise} \end{cases}$$

C_i = capacity of existing link i

c_i = capacity per unit width of link i

Let the traffic demand G be a parameter. The traffic volumes of O-D pair Y_r^k and the improvement made to each link x_i can be obtained by calculating a parametric linear programming (LP) while increasing G step by step. Then cuts with acceptable flow greater than the minimum cut are produced with accompanying increases in the road network capacity. Attempts are then made to search for the cuts from dual variables of dual problem. The dual problem of optimal network design problem is described as follows:

$$\text{maximize} \quad \sum_{k=1}^{q} p_k G \cdot y_k - \sum_{i=1}^{m} C_i w_i \tag{6}$$

subiject to

$$y_k - \sum_{i=1}^{m} {}_i\delta_r^k \cdot w_i \leqq 0 \quad \begin{Bmatrix} k=1,2,\cdots,q \\ r=1,2,\cdots,n_k \end{Bmatrix} \tag{7}$$

$$C_i\, w_i \leqq d_i \quad (i=1,2,\cdots,m) \tag{8}$$

$$w_i \leqq 0 \qquad (i=1,2,\cdot\cdot,m) \tag{9}$$

where

y_k = dual variable corresponding to the conservation equation of O-D pair k

w_i = dual variable corresponding to the capacity constraint ineqality of link i

According to the complementary theorem of LP[7], a relationship between slack variable λ_i and dual variable w_i given in Eq. (3) is obtained as shown in Eq. (10). From this equation, the two variables must be one of the following combinations.

$$\lambda_i \cdot w_i = 0 \quad (i=1,2,\cdots,m) \tag{10}$$

$$\left.\begin{array}{ll} \text{(i)} & \lambda_i=0, w_i>0 \\ \text{(ii)} & \lambda_i>0, w_i=0 \\ \text{(iii)} & \lambda_i=0, w_i=0 \end{array}\right\} \tag{11}$$

From the economic viewpoint of the dual variable, links choosen as bottleneck sections are contained in case (i) of Eq. (11). In other words, a set of links with a positive value for the dual variable makes a cut set which partitions the set of nodes into two exclusive sets. And the dual variables quantify the marginal changes to the primal objective function which result from unit changes to the capacity constraints[4]. Therefore it becomes possible to search for cuts with acceptable flow greater than the minimum cut by classifying each link having a similar influence on the primal objective function. Furthermore, the problem of determining which cut section each O-D pair would cross can be considered from dual variable y_k, because similar arguments, as in the case between Eq. (10) and (11), can be made between dual variable y_k and the slack variable given in Eq. (2).

2.2 Formulation of O-D cut matrix

Because the rank of the cut matrix (incident matrix of cut and link) is (n-1) in a network constituting n node, a search can only be made for (n-1) independent cuts even when using the LP method. When considering elementary cuts, the formulation of the O-D cut matrix carried out according to the following steps.

Step 1. Solve a parametric LP with the traffic demand G as a parameter and then search for (n-1) independent cuts using dual variable w_i

Step 2. Among cuts search in Step 1, determine which cut section each O-D pair would cross using dual variable y_k.

Step 3. Search for other elementary cuts from the combination of cuts explored in Step 1.

Step 4. Among cuts explored in Step 3, the problem of which cut section each O-D pair would

Table 1 Unit O-D traffic volumes and O-D pair numbers

O \ D	1	2	3	4	5	6	7	8	9	10
1		0.095	0.077	0.095	0.084	0.104	0.056	0.014	0.017	0.005
2	1		0.038	0.022	0.012	0.013	0.006	0.0	0.024	0.001
3	2	10		0.050	0.016	0.015	0.005	0.005	0.0	0.003
4	3	11	17		0.029	0.022	0.007	0.004	0.020	0.001
5	4	12	18	23		0.068	0.013	0.003	0.0	0.0
6	5	13	19	24	29		0.032	0.003	0.004	0.002
7	6	14	20	25	30	32		0.013	0.005	0.001
8	7	-	21	26	31	33	36		0.011	0.001
9	8	15	-	27	-	34	37	39		0.004
10	9	16	22	28	-	35	38	40	41	

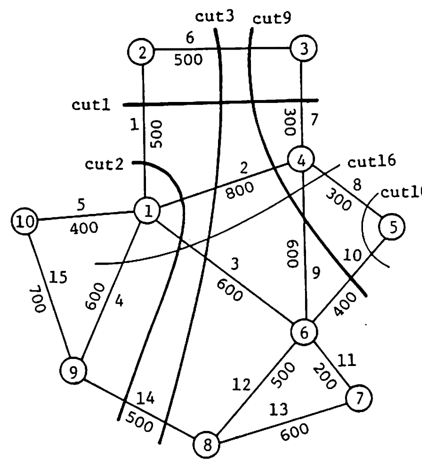

Fig. 1 Road network and independent cuts

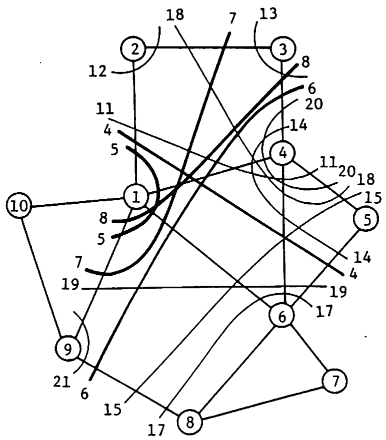

Fig. 2 Search for elementary cuts based on combinations of independent cuts

cross is solved referring to Step 2.

Step 5. The acceptable flow of all cuts explored in Step 1 and 3 is calculated from Eq. (12) and (13), rearrange cuts in order of acceptable flow (from small to large), and finally formulate the O-D cut matrix.

$$P_w = \sum_{k \in Q_w} p_k \tag{12}$$

$$F_w = \sum_{i \in R_w} C_i / P_w \tag{13}$$

where

P_w = sum of the unit O-D traffic volumes through cut w

Q_w = set of O-D pairs through cut w

R_w = set of links contained in cut w

Allowing for all combinations in Step 3, in case of w number of cuts, there exists 2^{w-1} different combinations; thus requiring a large number of calculations. However, by dividing the links constituing the road network into either those bordering on only the inner region or those in both inner and outer regions as well as by introducing graph theory, i. e., star without loop is an elementary cut, a search for elementary cuts can easily be conducted[3].

Table 2 Dual variable w_i for each traffic demand G

G \ link #	1	2	3	4	6	7	8	9	10	14
70000	0.025					0.025				
82500	0.042	0.017	0.017			0.025				0.017
85000	0.042	0.042	0.042		0.025	0.025				0.042
92500	0.042	0.059	0.042		0.042	0.025		0.017	0.017	0.042
107500	0.042	0.059	0.042		0.042	0.025	0.012	0.017	0.029	0.042
115000	0.042	0.059	0.046	0.004	0.042	0.025	0.012	0.021	0.033	0.042
140000	0.042	0.059	0.046	0.004	0.042	0.025	0.012	0.021	0.033	0.042

Table 3 Dual variable y_k for each traffic demand G

G \ O-D	1-2	1-3	1-4	1-5	1-6	1-7	1-8
70000	0.025	0.025					
82500	0.042	0.042	0.017	0.017	0.017	0.017	0.017
85000	0.042	0.067	0.042	0.042	0.042	0.042	0.042
92500	0.042	0.084	0.059	0.059	0.042	0.042	0.042
107500	0.042	0.084	0.059	0.071	0.042	0.042	0.042
115000	0.042	0.084	0.059	0.071	0.046	0.046	0.046
140000	0.042	0.084	0.059	0.071	0.046	0.046	0.046

Table 4 Capacity of the cut, sum of the unit O-D traffic volumes through the cut, and acceptable flow

cut #	C_w	P_w	F_w
1	24000	0.344	69767
2	48000	0.598	80267
3	48000	0.569	84358
4	48000	0.561	85561
5	48000	0.554	86642
6	48000	0.542	88561
7	48000	0.525	91429
8	48000	0.498	96385
9	48000	0.494	97166

cut \ link	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0
2	1	1	1	0	0	0	0	0	0	0	0	0	0	1	0
3	0	1	1	0	0	1	0	0	0	0	0	0	0	1	0
4	1	1	0	0	0	0	0	0	1	1	0	0	0	0	0
5	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
6	0	1	1	0	0	0	1	0	0	0	0	0	0	1	0
7	0	1	1	1	0	1	0	0	0	0	0	0	0	0	0
8	0	1	1	1	0	0	1	0	0	0	0	0	0	0	0
9	0	1	0	0	0	1	0	0	1	1	0	0	0	0	0

Fig. 3 Cut matrix C

2.3 Numerical example

To illustrate the discussion in the previous Paragraph, let us consider the road network depicted in Fig. 1. Table 1 lists the unit O-D traffic volumes (upper right half) and O-D pair number (lower left half). Figures given above links in Fig. 1 are link number and link length (m), respectively. Let the number of traffic lanes of each link be one and the capacity of each link 12000. Three to four paths are choosen as possible travel routes of each O-D pair, including the shortest path.

Calculation of road network capacity gives 69767 according to cut 1 (minimum cut) constituting link 1 and 7 of Fig. 1. Then as a result of parametric LP, dual variables w_i and y_k obtained for each

cut \ O-D	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41
1	1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	1	1	1	1	1	1	0	0	0	0	0	0	0	1	1	0	0	0	0	0	1	0	0	0	0	1	1	0	0	0	0	0	1	1	0	1	1	1	1	0
3	0	1	1	1	1	1	1	0	0	1	1	1	1	1	0	0	0	0	0	0	0	1	0	0	0	0	1	1	0	0	0	0	0	1	1	0	1	1	1	1	0
4	1	1	1	1	0	0	0	0	0	0	0	0	1	1	1	1	0	0	1	1	1	1	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
5	1	1	1	1	1	1	1	0	0	0	0	0	0	1	1	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	1	1	1	1	1	0	0	0	1	1	1	1	0	0	1	1	1	1	1	0	0	0	0	0	1	1	0	0	0	0	0	1	1	0	1	1	1	1	0
7	0	1	1	1	1	1	1	0	0	1	1	1	1	1	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	1	1	1	1	1	0	0	0	1	1	1	1	0	0	1	1	1	1	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	1	1	1	0	0	0	0	0	1	1	1	0	0	0	0	0	0	1	1	1	1	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0

Fig. 4 O-D cut matrix K

traffic demand G and shown in Table 2 and 3, respectively. Table 3 only shows O-D pairs involving node 1. A search is conducted for the 6 independent cuts as shown in Fig. 1 according to changes in dual variable in Table 2. Furthermore, a search is made for other elementary cuts based on the combination of 6 independent cuts (Fig. 2).

In regard to these elementary cuts, the capacity of the cut (C_w), sum of the unit O-D traffic volumes through the cut (P_w), and acceptable flow of the cut (F_w) are shown in Table 4. By putting the cuts in order of acceptable flow (from small to large), the cut matrix C and O-D cut matrix K are obtained (Fig. 3 and 4). Among the 21 cuts investigated, only 9 cuts-with an acceptable flow smaller than 100000 and which are also required for the analysis as given in latter Section-are illustrated : the 9 cuts are drawn as broad lines in Fig. 1 and 2. The O-D pair number in Fig. 4 corresponds to the lower left half of Table I.

3. SENSITIVITY ANALYSIS BY O-D CUT MATRIX

Various TSM alternatives are, in other words, attempts to increase road network capacity encouraging changes in the O-D traffic pattern. Therefore, we shall consider effects on the road network capacity brought about by changes in the O-D traffic pattern. A sensitivity analysis is carried out by calculating the road network capacity accompanying changes in the O-D traffic pattern via simple matrix operation based on the proposed O-D cut matrx.

Now, let w be the number of cuts to be made in the calculation. Then, the steps required to calculate the road network capacity accompanying changes in the O-D traffic pattern are as follows :

Step 1. Let L be column vector for the capacity of each link (Eq. (14)). The capacity of each cut M_w is obtained by multiplying column vector L by cut matrix C. Where L is the $M \times 1$ column vector, C is the $w \times m$ matrix and superscript t denotes matrix transposition.

$$L = [L_1, L_2, \cdots, L_l, \cdots, L_m]^t \tag{14}$$

$$M = C \cdot L \tag{15}$$

Step 2. Let p be column vector for the unit O-D traffic volumes (Eq. (16)). Replace each element of column vector p by the unit O-D traffic volumes resulting from changes in the O-D traffic pattern due to various TSM alternatives. Then, the sum of the unit O-D traffic volumes through the cut (P_w) is obtained by multiplying column vector p by the O-D cut matrix K. Where p is the $r \times 1$ column vector, P is the $w \times 1$ column vector and K is the $w \times q$ matrix.

$$p = [p_1, p_2, \cdots, p_k, \cdots, p_r]^t \tag{16}$$

$$P = [P_1, P_2, \cdots, P_w, \cdots, P_w] \tag{17}$$

Step 3. Substitute the inverse values of the P_w elements of column vector P for the diagonal elements of matrix D. Where D is the $W \times W$ matrix and $D_{ww} = 1/P_w$.

	2	22	1	15	16	17	11	18	19	13	12	14	20	21	3	4	28	27	5	6	7	39	37	34	35	38	40	10	29	24	30	25	26	31	8	9	23	32	33	36	41
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	1	1	0	0	0	0	1	0	0	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
4	1	1	1	1	1	0	0	0	1	1	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0
5	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	1	1	0	0	0	0	1	0	0	1	1	1	0	0	1	1	1	0	1	1	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	1	1	0	0	0	0	1	0	1	0	1	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0

Fig. 5 O-D cut matrix K′

$$D=\begin{array}{c} 1 \\ 2 \\ \vdots \\ \vdots \\ W \end{array}\left(\begin{array}{ccc} 1 & 2 \cdots\cdots & W \\ D_{11} & & \\ & D_{22} & 0 \\ & 0 & \\ & & D_{WW} \end{array}\right) \tag{18}$$

Step 4. Calculation the acceptable flow of each cut (F_w) by multiplying column vector M by matrix D. Where F is the $w \times 1$ column vector.

$$F = D \cdot M \tag{19}$$

Step 5. The road network capacity RN accompanying changes in the O-D traffic pattern can be obtained using a cut with the minimum acceptable flow.

$$RN = \min \{F_1, F_2, \cdots, F_w\} \tag{20}$$

These procedures are based on the assumption that changes in the unit O-D traffic volumes alters the acceptable flow of the cut which the O-D pair in question crosses; thus also changing the road network capacity. A sensitivity analysis of the accompanying changes in the O-D traffic pattern can be accomplished by compairing the capacity of the existing road network and RN of Eq. (20).

4. APPLICATION TO SOME TRAFFIC MANAGEMENT PROBLEMS

Using the calculation procedures discussed in the previous Section, it is possible to consider influence of the road network capacity brought about by various TSM alternatives. However, in trying to increase the road network capacity, detailed arguments have not yet been presented concerning which O-D pairs and zones must be restricted. Now, consideration is given to the identification of which O-D pairs subjeted to restrictions will result in an increase of road network capacity by classifying O-D pairs under restrictions into two categories. In both cases, the study is carried out using the O-D cut matrix.

4.1 Restrictions imposed on specific O-D pairs

Various TSM alternatives with restrictions imposed on specific O-D pairs are considered to include the followings: diverting the travel mode to other means, efficiency of car usage by carpool or vanpool, etc. Now, the investigation will be conducted using O-D cut matrix K′ (Fig. 5) obtained by rearranging each O-D pair of matrix K (Fig. 4)). This O-D cut matrix K′ is obtained by arranging the order of acceptable flow of the cut through which each O-D pair crosses. Matrix K′ takes into account the consideration that unless restrictions are imposed on the O-D pairs which cross the cut with smaller acceptable flow (including the minimum cut), no influence can be experted upon the road network capacity. Therefore, this matrix is equivalent to that obatained by rearranging each O-D pair in order of effects on increases of road network capacity.

Table 5 Road network capacity by restriction on specific O-D pairs

O-D pair		1-3	1-2	3-4	1-2 1-5	2-4 1-5
Unit O-D traffic volumes		0.077	0.095	0.050	0.095 0.084	0.022 0.084
O-D pair numbers crossing each cut	1	1	1	1	1	1
	2	1	1	0	2	1
	3	1	0	0	1	2
	4	1	1	0	2	1
	5	1	1	0	2	1
	6	0	0	1	1	2
	7	1	0	0	1	2
	8	1	0	1	1	2
	9	1	0	0	1	2
F'_1 a)		89887	96385	81632	96385	74534
F'_w b)		88561 (6)	84358 (3)	80267 (2)	106667 (10)	106667 (10)
Traffic volumes restricted		6819	8014	4013	17252	7900
Road network capacity		81742	76344	76254	79133	66634

a) F'_1 = acceptable flow of cut 1 by restrictions imposed on specific O-D pairs

b) F'_w = acceptable flow which can be achieved by restrictions and its cut number w (figure in parentheses)

For example, when attempting to increase the road network capacity by imposing restrictions independently on each O-D pair, among the O-D pairs shown in Fig. 5, 14 O-D pairs from O-D 1-3 (2) to O-D 3-8 (21) which cross the minimum cut must be subjected to restrictions: figures in parentheses reprsent an O-D pair number in Fig. 5. However, also concerning theses O-D pairs, as shown in example calculations in Table 5, a possible upper limit of the capacity expansion will differ depending on the acceptable flow which can be achieved under the restrictions. Whereas, with regard to the 27 O-D pairs from O-D 1-4 (3) on, it is impossible to increase the road network capacity independently. Therfore, these O-D pairs must be resricted in combination with the O-D pairs crossing the minimum cut such as with O-D 1-5 (4) as shown in Table 5.

In Table 5, for reasons of simplification, it is considered that the O-D pair which is subjected to restrictions is not able to generate. The road network capacity is calculated by subtracting the traffic volumes restricred from the acceptable flow F'_w which can achieved under the restrictions. As stated above, various TSM alternatives aim at increasing the road network capacity by encouraging changes in the O-D traffic pattern. However, there are cases such as O-D 2-4 and 1-5 in Table 5, when the acceptable flow which can be achieved by restrictions in larger, but the capacity expansion can't be expected.

4.2 Restriction imposed on zones

This method, restriction imposed on zones, restricts, at a constant rate, all O-D pairs generating in and attracting to the identical zone by such means as parking restriction, road pricing, and

Table 6 Sum of the unit O-D traffic volumes of O-D pairs through each cut for every zone

Node #		1	2	3	4	5	6	7	8	9	10
Z_i a)		0.547	0.211	0.209	0.250	0.225	0.263	0.138	0.054	0.085	0.018
$_wP_i$ b)	1	0.172	0.173	0.171	0.072	0.028	0.028	0.011	0.005	0.024	0.004
	2	0.525	0.120	0.080	0.116	0.084	0.110	0.062	0.026	0.064	0.009
	3	0.430	0.091	0.118	0.138	0.096	0.123	0.068	0.026	0.040	0.008
	4	0.351	0.139	0.105	0.149	0.168	0.118	0.031	0.012	0.020	0.004
	5	0.525	0.120	0.080	0.096	0.084	0.104	0.056	0.014	0.024	0.005
	6	0.353	0.053	0.091	0.188	0.112	0.138	0.073	0.031	0.040	0.005
	7	0.430	0.091	0.118	0.118	0.096	0.117	0.062	0.014	0.0	0.004
	8	0.353	0.053	0.091	0.168	0.112	0.132	0.067	0.019	0.0	0.001
	9	0.256	0.072	0.143	0.171	0.180	0.105	0.025	0.012	0.020	0.004

a) Z_i = sum of the umit O-D traffic volumes of O-D pairs generating in and attracting to each zone

b) $_wP_i$ * sum of the unit O-D traffic volumes of O-D pairs through each cut for every zone

Table 7 Road network capacity by restriction imposed on zones

Zone #	1	2	3	4	5	6	7
Z_i	0.547	0.211	0.209	0.250	0.225	0.263	0.138
$Z_i - Z'_i$ a)	0.447	0.111	0.109	0.150	0.125	0.163	0.038
F' b)	76786 (1)	88724 (2)	85714 (2)	76142 (1)	72386 (1)	71995 (1)	71422 (1)
$_wP_i$ → $_wP_i - {_wP'_i}$ c)	0.172 ↓ 0.141	0.120 ↓ 0.063	0.080 ↓ 0.042	0.072 ↓ 0.043	0.028 ↓ 0.016	0.028 ↓ 0.017	0.011 ↓ 0.003
$F' \times Z'_i$ d)	7679	8872	8571	7614	7239	7200	7142
Road network capacity	69107	79852	77143	68528	65147	64795	64280

a) $Z_i - Z'_i$ = sum of the unit O-D traffic volumes which can be generated in and attracted to zone i by restrictions

b) F' = minimum acceptable flow and its cut number (figure in parentheses)

c) $_wP_i - {_wP'_i}$ = sum of the unit O-D traffic volumes which cross the cut with the minimum acceptable flow

d) $F' \times Z'_i$ = traffic volumes which are restricted staggered working hours. In this case, since many O-D pairs which are generating in and attracting to the identical zone are subject to restrictions, it is difficult to determine which zones should be restricted to increase the road network capacity. Based on the O-D cut matrix in Fig. 4, we can find out which cut each O-D pair generating in and attracting to the identical zone would cross ; then, for every zone, the sum of the unit O-D traffic volumes of the O-D pairs crossing each cut is calculated. A summary of these results is shown in Table 6.

For example, the sum of the unit O-D traffic volumes of O-D pairs crossing cut 1 of zone 1 (0.172) can be obtained from O-D 1-2 (0.095) and O-D 1-3 (0.077) from among the O-D pairs generating in and attracting to zone 1. Table 6 is formulated considering that even in the zone restriction method, zones in which greater number of O-D pairs cross the cut with smaller acceptable flow (including the minimum cut), have larger effects on increasing the road network capacity. As a result, in this case, zones 1, 2 and 3 having larger sums of unit O-D traffic volumes for O-D pairs crossing cut 1 indicate higher influence on expanding the road network capacity than other zones.

Next, the road network capacity RN accompanying restrictions on each zone can be obtained by first calculating, using Eq. (21) and (22), the acceptable flow of each cut brought about by restrictions and then comparing these acceptable flows using Eq (23) an (24).

$$ {}_wP'_i = {}_wP_i \frac{Z'_i}{Z_i} \quad (w=1,2,\cdots,W) \tag{21} $$

$$ F'_w = M_w / ({}_wP_i - {}_wP'_i)(w=1,2,\cdots,W) \tag{22} $$

$$ F' = \min\{F'_1, F'_2, \cdots, F'_w\} \tag{23} $$

$$ RN = F'(1-Z'_i) \tag{24} $$

where

Z_i = sum of the unit O-D traffic volumes of O-D pairs generating in and attracting to zone i

Z'_i = sum of the unit O-D traffic volumes of O-D pairs restricted in zone i

${}_wP_i$ = sum of the unit O-D traffic volumes of O-D pairs crossing cut w from among the O-D pairs generating in and attracting to zone i

${}_wP'_i$ = sum of the unit O-D taffic volumes of O-D pairs restricted in cut w from among the O-D pairs restricted in zone i

Table 7 is the summary of results obtained when restricting, by 0.100, the sum of the unit O-D traffic volumes of O-D pairs generating in and attracting to each zone. Except for zone 2 and 3, the increase of road network capacity can't be expected to be achieved. This is because resrictions imposed on zones may also retrict many other O-D pairs unneccessarily from the viewpoint of road network capacity. Therefore, in zones which have larger sums of unit O-D taffic volumes of the O -D pairs crossing the minimum cut, expansion can be achieved to some extent, but much can't be expected for other zones.

INCREASING ROAD NETWORK CAPACITY BY TRANSFERRING URBAN FACILITIES

Another application attempted is an assessment of the effects on road network capacity of relocation of land-use activities by transferring urban facilities to another zone. In this analysis, transferring urban facilities such as public facilities, goods transport facilities and offices only between two zonal areas is disucussed. Transferring urban facilities causes changes in the traffic volumes of the O-D pairs generating in and attracting to the zones concerned as well as encouraging changes in the O-D traffic pattern. Thus, when increasing the road network capacity by transferring urban facilities, attention is paid to changes in the O-D traffic pattern.

zone \ zone	1	2	3	4	5	6	7	8	9	10
1	0	1	1	0	0	0	0	0	0	0
2	1	0	0	1	1	1	1	1	1	1
3	1	0	0	1	1	1	1	1	1	1
4	0	1	1	0	0	0	0	0	0	0
5	0	1	1	0	0	0	0	0	0	0
6	0	1	1	0	0	0	0	0	0	0
7	0	1	1	0	0	0	0	0	0	0
8	0	1	1	0	0	0	0	0	0	0
9	0	1	1	0	0	0	0	0	0	0
10	0	1	1	0	0	0	0	0	0	0

Fig. 6 Matrix U_1

zone \ zone	1	2	3	4	5	6	7	8	9	10
1	0	-1	-1	0	0	0	0	0	0	0
2	1	0	0	1	1	1	1	1	1	1
3	1	0	0	1	1	1	1	1	1	1
4	0	-1	-1	0	0	0	0	0	0	0
5	0	-1	-1	0	0	0	0	0	0	0
6	0	-1	-1	0	0	0	0	0	0	0
7	0	-1	-1	0	0	0	0	0	0	0
8	0	-1	-1	0	0	0	0	0	0	0
9	0	-1	-1	0	0	0	0	0	0	0
10	0	-1	-1	0	0	0	0	0	0	0

(1) V_1

	1	2	3	4	5	6	7	8	9	10
1	0	1	1	1	1	1	1	1	0	0
2	-1	0	0	0	0	0	0	0	1	1
3	-1	0	0	0	0	0	0	0	-1	1
4	-1	0	0	0	0	0	0	0	-1	-1
5	-1	0	0	0	0	0	0	0	-1	-1
6	-1	0	0	0	0	0	0	0	-1	-1
7	-1	0	0	0	0	0	0	0	-1	-1
8	-1	0	0	0	0	0	0	0	-1	-1
9	0	-1	1	1	1	1	1	1	0	0
10	0	-1	-1	1	1	1	1	1	0	0

(2) V_2

	1	2	3	4	5	6	7	8	9	10
1	0	0	1	1	1	1	1	1	0	0
2	0	0	1	-1	-1	-1	-1	-1	0	0
3	-1	-1	0	0	0	0	0	0	1	1
4	-1	-1	0	0	0	0	0	0	1	1
5	-1	1	0	0	0	0	0	0	-1	-1
6	-1	1	0	0	0	0	0	0	-1	-1
7	-1	1	0	0	0	0	0	0	-1	-1
8	-1	1	0	0	0	0	0	0	-1	-1
9	0	0	-1	-1	1	1	1	1	0	0
10	0	0	-1	-1	1	1	1	1	0	0

(3) V_3

Fig. 7 Inter-zonal sensitivity matrix V_w for cut 1, 2 and 3

In case of facility transfer, the unit O-D traffic volumes of O-D pairs change accordingly. Therefore, there are cuts which may decrease the acceptable flow because of an increase in the sum of the unit O-D traffic volumes crossing the cut. So, matrix U_w, expressing whether each O-D pair may cross the affected cuts, is first formulated based on the O-D cut matrix. Furthermore, from matrix U_w, inter-zonal sensitivity matrix V_w is formulated for each cut, while closely examining if facility transfer decreases the acceptable flow of the cut as aforementioned.

Then, study of a possible increase in the road network capacity accompanying facility transfer is performed as follows using matrix V_w.

Step 1. Formulate matrix U_w for each cut based on the O-D cut matrix. Where ${}_{ij}U_w$, an element of matrix U_w, is 1 when O-D pair ij crosses the cut and 0 otherwise. When ${}_{ij}U_w$ is 1, facility transfer from i to j zone may possible increase the acceptable flow of cut w.

Step 2. Calculate changes in the sum of the unit O-D traffic volumes of O-D pairs crossing cut w, ${}_{ij}P_w$, accompanying facility transfer. Where zone pairs which are 1 in matrix U_w are only calculated.

$$ {}_{ij}P_w=\frac{Z_{ij}}{Z_i}\left(\sum_{r\in {}_iP_w^0} P_r-\sum_{r\in {}_iP_w^1} P_r\right) \tag{25} $$

where

${}_iP_w^0$ = set of O-D pairs taking 0 in row i of matrix U_w

${}_iP_w^1$ = set of O-D pairs taking 1 in row i of matrix U_w

Z_{ij} = unit O-D traffic volumes decreasing in zone i as a result of transferring facilities zone i to j

	1	2	3	4	5	6	7	8	9	10
1		●	●	–	–	–	–	–	–	–
2	○		○	○	○	○	○	○	◎	◎
3	○	▲		○	○	○	○	○	○	○
4	▲	●	●		–	–	–	–	▲	▲
5	▲	●	●	–		–	–	–	▲	▲
6	▲	●	●	–	–		–	–	▲	▲
7	▲	●	●	–	–	–		–	▲	▲
8	▲	●	●	–	–	–	–		▲	▲
9	–	●	●	▲	▲	▲	▲	▲		–
10	–	●	●	▲	▲	▲	▲	▲	–	

Fig. 8 Summary of the patterns beloning to the facility transfer between each zone pair

Table 8 Pattern of combinations taken by each zone pair

pattern	example of zone pair belonging to pattern	Value of element in matrix V_1, V_2, V_3			Symbol of the pattern used in Fig. 8
		V_1	V_2	V_3	
1	2-9	1	1	0	◎
	3-4	1	0	0	○
2	2-1	1	-1	0	△
	2-4	1	0	-1	
	3-1	1	-1	-1	
	3-9	1	-1	1	
3	4-1	0	-1	-1	▲
	3-2	0	0	-1	
	4-9	0	-1	1	
	9-4	0	1	-1	
4	4-3	-1	0	0	●
	4-2	-1	0	1	
	1-2	-1	1	0	
	9-3	-1	1	-1	
	1-3	-1	1	1	
	10-3	-1	-1	-1	
5	1-4	0	1	1	–
	4-5	0	0	0	

Step 3. Reset respective elements of matrix V_w to -1 when ${}_{ij}P_w$ is positive and 1 when negative; thus formulating the inter-zonal sensitivity matrix V_w. When ${}_{ij}V_w$ is 1, facility transfer from i to j zone increases the acceptable flow of cut w, and when it is -1 it decreases the flow.

Step 4. Conduct Steps (1)-(3) for all cuts included in the calculation. Through inter-zonal sensitivity matrix V_w formulated for each cut, an assessment of the effects of transferr-

Table 9 Road network capacity by facility transfer between several zone pairs

Zone pair			2-9	3-4	2-1	4-1	4-3
Pattern to which zone pair in question belongs			1	1	2	3	4
$Z_i - Z_{ij}$ a)	i		0.211 ↓ 0.123	0.209 ↓ 0.134	0.211 ↓ 0.157	0.250 ↓ 0.187	0.250 ↓ 0.169
$Z_j + Z_{ij}$	j		0.085 ↓ 0.173	0.250 ↓ 0.325	0.547 ↓ 0.601	0.547 ↓ 0.610	0.209 ↓ 0.290
${}_{ij}P_w$ & ${}_{ij}F_w$ b)	1	0.344 69767	0.292 82192	0.305 78688	0.326 73620	0.344 69767	0.407 58968
	2	0.598 80267	0.596 80587	0.598 80267	0.630 76190	0.645 74419	0.598 80267
	3	0.569 84358	0.569 84358	0.569 84358	0.569 84358	0.598 80267	0.569 84358
Road network capacity			80587	78688	73620	69767	58968

a) $Z_i - Z_{ij}$ ($Z_j + Z_{ij}$) = variations of the sum of unit O-D traffic volumes by facility transfer from i zone to j zone

b) ${}_{ij}P_w$, ${}_{ij}F_w$ = sum of the unit O-D traffic volumes which cross the cut and the acceptable flow of the cut by facility transfer

ing urban facilities on the road network capacity is carried out.

Matrix U_1 (Fig. 6), which expresses whether each O-D pair crosses cut 1 is based on the O-D cut matrix in Fig. 4. Furthermore, an inter-zonal sensitivity matrix for each cut is formulated according to the above mentioned Steps (Fig. 7). Cuts up to number 3 are considered when facility transfer is limited to those between two zonal areas. In the three sensitivity matrices, many different combinations can be taken by each zone pair ; however, when considering the assessment of effects on the road network capacity, they can be classified into five pattern as illustrated in Table 8.

The assessment of effects of each pattern on the road network capacity is as follows

Pattern 1. Zone pair where the increase of road network capacity doesn't exceed the acceptable flow of a certain cut (symbol ◎ up to cut 3, symbol ○ up ho cut 2).

Pattern 2. Zone pair where the increase of road network capacity is restricted to some extent : the acceptable flow of cut 1 is increased but the flow of cut 2 or 3 is decreased (△).

Pattern 3. Zone pair where facility transfer exceeding a certain extent reduces road network capacity : no influence is exerted on the acceptable flow of cut 1, but the flow of cut 2 and 3 is decreased (▲).

Pattern 4. Zone pair where the road network capacity is decreased because the acceptable flow of cut 1 is decreased (●).

Pattern 5. Zone pair without any influence on road network capacity (—).

Fig. 8 is the summary of the patterns belonging to the facility transfer between each zone pair. In this example, it is possible to increase the road network capacity only with facility transfer from zone 2 or 3 to other zones. Table 9 results from the assumption of facility transfer between several zone pairs : the calculation is made taking into account trasfers equivalent to 0.100 of the unit O-D traffic volumes between two zonal areas.

CONCLUSIONS

In the present study, the method of formulating the O-D cut matrix based on the complementary theorem of LP is investigated. Furthermore, the effects of various TSM alternatives on the road network capacity are analyzed by a sensitivity analysis based on the O-D cut matrix. Major findings are as follows.

(a) Various TSM alternatives encourage changes in the O-D traffic pattern. Therefore, the road network capacity accompanying changes in the O-D traffic pattern can be also be evaluated by simple matrix algebra based on the O-D cut matrix.

(b) When attempts are made to restrict any specific O-D pair, road network capacity can be easily estimated by formulating O-D cut matrix K′ obtained by rearranging each O-D pair in order of effects on the increase of road network capacity.

(c) When zones are subject to restrictions, road network capacity can be easily estimated by knowing which cut the O-D pairs generating in and attracting to the identical zone would cross and calculating the sum of the unit O-D traffic volumes of the O-D pairs crossing the cut.

(d) Furthermore, when the transfer of urban facilities is carried out between two zonal areas, the road network capacity can be evaluated by formulating an inter-zonal sensitivity matrix for each zone.

In the future, research will be conducted concerning the extent various TSM alternatives influence on the O-D traffic pattern. Since enlargements of the road network rapidly increase the number of O-D pairs and cuts, the study will include applications to a large scale road network.

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