Elastic Deformation of Semi-Infinite Medium by Boundary Element Method

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Abstract

The boundary element integral formulation and its numerical implementation for 2−dimensional semi-infinite problem without cavities are reviewed to present the detailed forms of relevant matrices which are not published explicitly up to now. Two examples, then, are presented to show the excellence of the method in engineering practice.

1. Notation

u_j : displacement vector
p_j : traction vector
σ_{ij} : stress tensor
b_j : body force vector
ε_{ij} : total strain tensor
u^*_{ij}, p^*_{ij} : tensors corresponding to the fundamental solutions
E, G : Young's and shear moduli
ν : Poisson's ratio
Γ : boundary of the body
Ω : domain of the body
δ_{ij} : Kronecker delta
S, s : load point for boundary and domain
Q, q : field point for boundary and domain
s' : image of s with respect to surface
r : distance between s and q
R : distance between s' and q
n_j : outward normal vector
ξ, η : coordinate of inner points
$|J|$: Jacobian of transformation
η_k : local coordinate associated with boundary element
ℓ : length of element
N_1, N_2 : interpolation functions
L : the number of boundary elements

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n : the number of boundary nodes
K : the number of Gaussian integration points
W_k : weighting factor
H : system matrix of all coefficients C_{ij}
U : system matrix of all displacements in the domain
U^* : matrix of displacement fundamental solutions
G : system matrix of all displacement fundamental solutions
P : system matrix of all tractions
I : unit diagonal matrix
A : system matrix of unknown boundary conditions
x : system matrix of all unknown boundary values
f : system matrix of all prescribed values
D : system matrix to obtain stresses

2. Introduction

Boundary element method (or boundary integral equation method) is the method that the governing differential equation is transformed into the integral equation defined over the surface and then solved numerically by discretizing the boundary into a number of elements. This allows the final system of equations even smaller than that of finite element method.

Followings are important features of this method ;

1) the dimension of the problem can be reduced by one
2) data preparation is simple to solve the problem
3) calculations on arbitrary internal points can be carried out easily and accurately
4) infinite or semi-infinite problems are properly modeled.

Above characteristics increase the range of problem analyzed and decrease computing time and cost. This would be of much greater advantage than finite element method as applied to more complicated problems.

On semi-infinite problems, few publications are found to attempt to present the detailed forms of matrices with proper definition and results of differential process of parameters used. In this paper an outline of boundary integral formulation is first given using fundamental solutions for elastostatic half-plane type problem. The numerical implementation is then presented and some examples are compared with analytical results.

3. Boundary Integral Formulation

Basic theory of boundary element formulation for 2-D semi-infinite problem is briefly reviewed in this section.

3. 1 Semi-Infinite Body

Static equilibrium of forces and moments of infinitesimal element within the body gives following equilibrium equation

$$\sigma_{ij,i} + b_i = 0 \tag{1}$$

where first term represents space derivatives of stress tensor component and second stands for the component of body force. Summation convention of tensor (Cartesian tensor notation) is used throughout this paper avoiding long repetition of an index and the partial derivative is denoted by a comma.

Strains can be represented by the Cauchy's infinitesimal strain tensor neglecting the square and product of derivatives

$$\varepsilon_{ij}=\frac{1}{2}(u_{i,j}+u_{j,i}). \qquad (2)$$

For an isotropic elastic material, Hooke's Law is stated in the form

$$\sigma_{ij}=2G\varepsilon_{ij}+\frac{2G\nu}{1-2\nu}\varepsilon_{kk}\delta_{ij}. \qquad (3)$$

The equilibrium condition is expressed in terms of displacement by substituting equation (2) into (3) and then the result into equation (1). This is well-known Navier equation written in the following form

$$Gu_{j,kk}+\frac{G}{1-2\nu}u_{k,kj}+b_j=0. \qquad (4)$$

It is the object here to obtain the solution of equation (4) for 2-D semi-infinite body by the direct boundary element method.

The integral equation should be deduced either from the weighted residual technique or from Betti's second reciprocal work theorem using the Dirac delta function. The result is in the form

$$u_i(s)=\int_\Gamma u^*_{ij}(s,\ Q)p_j(Q)d\Gamma-\int_\Gamma p^*_{ij}(s,\ Q)u_j(Q)d\Gamma+\int_\Omega u^*_{ij}(s,\ q)b_j(q)d\Omega \qquad (5)$$

which relates displacements at inner point s with tractions and displacements on boundary (at Q). $u^*_{ij}(s,\ Q)$ and $p^*_{ij}(s,\ Q)$ represent the displacements and tractions in the j direction at point Q (on boundary) due to a unit point force acting in the i direction applied at point s (in domain). The last integral on the right-hand-side is the body force term which is evaluated as domain integral. Body force integral, however, is not considered through this paper.

u^*_{ij} and p^*_{ij} are called fundamental solution. Fundamental solution for semi-infinite problem is as follows (load point s on the boundary in Fig. 1)

$$\begin{aligned}
u^*_{11}&=-\frac{1}{2\pi G}\{2(1-\nu)\ln r-r_{,1}^2\}\\
u^*_{12}&=-\frac{1}{2\pi G}\{(1-2\nu)\theta-r_{,1}r_{,2}\}\\
u^*_{21}&=-\frac{1}{2\pi G}\{-(1-2\nu)\theta-r_{,2}r_{,1}\}\\
u^*_{22}&=-\frac{1}{2\pi G}\{2(1-\nu)\ln r-r_{,2}^2\}
\end{aligned} \qquad (6)$$

where

$$\theta=\tan^{-1}(R_2/R_1)$$

$$r_{,i}=\frac{\partial r}{\partial x_i}=\frac{r_i}{r} \qquad (7)$$

and

$$p^*_{ij}=-\frac{2}{\pi r}\{r_{,i}r_{,j}\frac{\partial r}{\partial n}\}. \qquad (8)$$

The limiting form of equation (5) as the inner point s moved to boundary point S must be examined to obtain a boundary integral equation for unknown boundary values when boundary conditions are prescribed. Interpreting the integrals which present singularities on surface in the sense of Cauchy principal value and considering the condition of continuity of boundary displacement u, the following equation arises[1] (body force term omitted)

$$C_{ij}(S)u_j(S)=\int_\Gamma u^*_{ij}(S,Q)p_j(Q)d\Gamma-\int_\Gamma p^*_{ij}(S,Q)u_j(Q)d\Gamma \tag{9}$$

where the second integral on the right-hand-side is to be taken in the Cauchy principal sense. The coefficient $C_{ij}(S)$ is diagonal matrix which generally depends on the continuity of surface. But it will be shown later that the coefficient can be obtained indirectly by applying the rigid body movement representation.

Considering the behaviour of each term of equation (9) at infinity i.e. evaluating the asymptotic behaviour by examining the order of variables, boundary integral equation for semi-infinite regions without cavities (e. g. tunnels) is simply represented by

$$C_{ij}(S)u_j(S)=\int_\Gamma u^*_{ij}(S,Q)p_j(Q)d\Gamma \tag{10}$$

where the integral is defined on loaded boundary only. Expression (10) is very useful as applied to the semi-infinite problem with partial surface loads.

From equation (5) and above derivation, the representation of displacements at inner points comes to

$$u_i(s)=\int_\Gamma u^*_{ij}(s,Q)p_j(Q)d\Gamma. \tag{11}$$

Stresses at inner points are obtained by substituting the derivatives of equation (11) into the Cauchy's strain tensor (2) and then the results into the Hooke's Law (3). The expression is as follows

$$\sigma_{ij}(s)=\int_\Gamma D^*_{ijk}(s,Q)p_k(Q)d\Gamma \tag{12}$$

where the tensor D^*_{ijk} is derived from the differentiations of u^*_{ij} with respect to the coordinates of inner point s.

3. 2 Numerical Implementation

Boundary integral equation (10) can be written in the discretized matrices form as follows

$$HU=\sum_{j=1}^{L}\left\{\int_{\Gamma j}U^*Nd\Gamma\right\}P^{(n)} \tag{13}$$

where the boundary is discretized into L elements (denoted Γ_j) with n nodes. N represent the interpolation functions which approximate boundary displacements and tractions over each elements. The integral of equation (13) is transformed to the local coordinates system since the interpolation functions N are usually expressed in terms of local coordinate. The transformation is

$$d\Gamma=|J|d\eta \tag{14}$$

where η stands for local coordinate and $|J|$ is the Jacobian of transformation.

Numerical integration schemes are effective procedures for the analytically difficult integrations. The integral in equation (13) can be written in the form

$$\int_{\Gamma j}U^*Nd\Gamma=\int_{-1}^{1}U^*N|J|d\eta=\sum_{k=1}^{K}|J|_k(U^*N)_kW_k \tag{15}$$

where K is the number of Gaussian integration points and W_k is the associated weighting factor.

Equation (13) can be written in the matrix form as

$$HU=GP. \tag{16}$$

Assuming a unit rigid body translation to the above equation, zero traction condition leads to following relation

$$HI=O \tag{17}$$

where I is a unit vector of displacement. Equation (17) gives computation of the diagonal elements of H as (summation not implied)

$$H_{ii}=-\sum_{\substack{j=1\\j\neq i}}^{n}H_{ij}\ (i=1,2,3,\cdots,n). \tag{18}$$

Above equation, however, is valid only for finite body. Considering the limiting behaviour again for the semi-infinite case, equation (18) is replaced by

$$H_{ii}=I-\sum_{\substack{j=1\\j\neq i}}^{n}H_{ij}\ (i=1,2,3,\cdots,n) \tag{19}$$

and it implies the coefficient C_{ij} in equation (10) can be evaluated indirectly.

By applying the prescribed boundary condition, equation (16) can be reordered to give the unknown boundary values

$$Ax=f \tag{20}$$

where A is fully populated matrix of order 2n, unknown displacements and tractions are included in x, and f is formed by the prescribed boundary values.

3. 3 Linear element

The discretization of the boundary by means of linear elements has been found to be efficiently accurate for the numerical computation[1]. Numerical implementation according to this element type will be shown in this section.

The Jacobian of transformation for linear element is

$$|J|=\ell/2 \tag{21}$$

where ℓ replesents the length of each element.

Interpolation functions are as follows

$$N_1=\frac{1}{2}(1-\eta),\quad N_2=\frac{1}{2}(1+\eta) \tag{22}$$

where subscripts denote the extremities of the element.

Applying equations (21) and (22) to the integral in equation (15) gives the representation of submatrix g of G in the form

$$g=\frac{\ell}{2}\int_{-1}^{1}U^*Nd\eta=\begin{bmatrix} g^{①}_{\mathrm{I\,I}} & g^{①}_{\mathrm{I\,II}} & g^{②}_{\mathrm{I\,I}} & g^{②}_{\mathrm{I\,II}} \\ g^{①}_{\mathrm{II\,I}} & g^{①}_{\mathrm{II\,II}} & g^{②}_{\mathrm{II\,I}} & g^{②}_{\mathrm{II\,II}} \end{bmatrix} \tag{23}$$

where superscripts denote the end nodes of the element and subscripts derive from those of fundamental solutions (6).

For the case of the field point q and the load point s being on the same element during the integration along the boundary, the half-plane fundamental solutions are singular because of the zero distance. Analytical computation, however, can be carried out in this case and the results are

$$g^{①}_{\mathrm{I\,I}}=\frac{\ell}{4\pi G}\left\{2(1-\nu)\left(\frac{3}{2}-\ell n(\ell)\right)+\cos^2\theta\right\}$$

$$g^{②}_{\mathrm{I\,I}}=\frac{\ell}{4\pi G}\left\{2(1-\nu)\left(\frac{1}{2}-\ell n(\ell)\right)+\cos^2\theta\right\}$$

$$g^{①}_{\mathrm{I\,II}}=\frac{\ell}{4\pi G}\left\{-(1-2\nu)\theta+\sin\theta\cos\theta\right\}$$

$$g^{②}_{\mathrm{I\,II}}=g^{①}_{\mathrm{I\,II}}$$

$$g^{①}_{\mathrm{II\,I}}=\frac{\ell}{4\pi G}\left\{(1-2\nu)\theta+\cos\theta\sin\theta\right\}$$

$$g^{②}_{\mathrm{II\,I}}=g^{①}_{\mathrm{II\,I}}$$

$$g^{①}_{\mathrm{II\,II}}=\frac{\ell}{4\pi G}\left\{2(1-\nu)\left(\frac{3}{2}-\ell n(\ell)\right)+\sin^2\theta\right\}$$

$$g^{②}_{\mathrm{II\,II}}=\frac{\ell}{4\pi G}\left\{2(1-\nu)\left(\frac{1}{2}-\ell n(\ell)\right)+\sin^2\theta\right\}. \tag{24}$$

And for the non-singular case, expressions are obtained in the numerical integration form

$$g^{①}_{\mathrm{I\,I}}=\frac{\ell}{2}\sum_{k=1}^{K}(u^*_{11}N_1)_k W_k=\frac{\ell}{8\pi G}\sum_{k=1}^{K}\left(-2(1-\nu)\ell n(r_k)+\cos^2\beta_k\right)(1-\eta_k)W_k$$

$$g^{①}_{\mathrm{I\,II}}=\frac{\ell}{2}\sum_{k=1}^{K}(u^*_{12}N_1)_k W_k=\frac{\ell}{8\pi G}\sum_{k=1}^{K}\left(-(1-2\nu)\theta+\cos\beta_k\sin\beta_k\right)(1-\eta_k)W_k$$

$$g^{①}_{\mathrm{II\,I}}=\frac{\ell}{2}\sum_{k=1}^{K}(u^*_{21}N_1)_k W_k=\frac{\ell}{8\pi G}\sum_{k=1}^{K}\left((1-2\nu)\theta+\sin\beta_k\cos\beta_k\right)(1-\eta_k)W_k$$

$$g^{①}_{\mathrm{II\,II}}=\frac{\ell}{2}\sum_{k=1}^{K}(u^*_{22}N_1)_k W_k=\frac{\ell}{8\pi G}\sum_{k=1}^{K}\left(-2(1-\nu)\ell n(r_k)+\sin^2\beta_k\right)(1-\eta_k)W_k$$

$$\cos\beta_k=(r_1/r)_k,\quad \sin\beta_k=(r_2/r)_k \tag{25}$$

where simply the interpolation function is replaced to N_2 of equation (22) as to around the node 2.

Once all boundary values obtained from equation (16) and (20), computation of displacements at internal points is also accomplished in the same way as the boundary values (equation (11)). No singularities are in those calculations since the fundamental solutions obviously have non-zero distances.

For the calculations of inner stresses, explicit forms can be represented through some differentiations. The matrix D in equation (12) for node 1 will be expressed by submatrix g of equation (25) as follows

$$D^{①}_{11}=\frac{2G(1-\nu)}{1-2\nu}\left\{g^{①}_{\mathrm{I\,I},\xi}+\frac{\nu}{1-\nu}g^{①}_{\mathrm{II\,I},\eta}\right\}$$

$$D^{①}_{12}=\frac{2G(1-\nu)}{1-2\nu}\left\{g^{①}_{\mathrm{I\,II},\xi}+\frac{\nu}{1-\nu}g^{①}_{\mathrm{II\,II},\eta}\right\}$$

$$D^{①}_{21}=G\left\{g^{①}_{\mathrm{I\,I},\eta}+g^{①}_{\mathrm{II\,I},\xi}\right\}=D^{①}_{12}$$

$$D^{①}_{22}=G\left\{g^{①}_{\mathrm{I\,II},\eta}+g^{①}_{\mathrm{II\,II},\xi}\right\}=D^{①}_{31}$$

$$D^{①}_{31}=\frac{2G(1-\nu)}{1-2\nu}\left\{\frac{\nu}{1-\nu}g^{①}_{\mathrm{I\,I},\xi}+g^{①}_{\mathrm{II\,I},\eta}\right\}$$

$$D^{①}_{32}=\frac{2G(1-\nu)}{1-2\nu}\left\{\frac{\nu}{1-\nu}g^{①}_{\mathrm{I\,II},\xi}+g^{①}_{\mathrm{II\,II},\eta}\right\} \tag{26}$$

where the superscripts and subscripts are the same as g and derivatives with respect to ξ and η are denoted by $,\xi$ and $,\eta$. And each derivatives are as follows

$$g^{①}_{\mathrm{I\,I},\xi}=-\frac{\ell}{8\pi G}\sum_{k=1}^{K}(1-\eta_k)W_k\left(2(1-\nu)\frac{\cos\beta_k}{r_k}-\frac{2\cos\beta_k\sin^2\beta_k}{r_k}\right)$$

$$g^{①}_{\mathrm{II\,I},\eta}=-\frac{\ell}{8\pi G}\Sigma(1-\eta_k)W_k\left((1-2\nu)\frac{\cos\beta_k}{r_k}+\frac{\cos\beta_k(2\sin^2\beta_k-1)}{r_k}\right)$$

$$g^{①}_{\mathrm{I\,II},\xi}=-\frac{\ell}{8\pi G}\Sigma(1-\eta_k)W_k\left((1-2\nu)\frac{\sin\beta_k}{r_k}+\frac{\sin\beta_k(2\cos^2\beta_k-1)}{r_k}\right)$$

$$g^{①}_{\mathrm{II\,II},\eta}=-\frac{\ell}{8\pi G}\Sigma(1-\eta_k)W_k\left(2(1-\nu)\frac{\sin\beta_k}{r_k}-\frac{2\sin\beta_k\cos^2\beta_k}{r_k}\right)$$

$$g^{①}_{\mathrm{I\,I},\eta}=-\frac{\ell}{8\pi G}\Sigma(1-\eta_k)W_k\left(2(1-\nu)\frac{\sin\beta_k}{r_k}+\frac{2\sin\beta_k\cos^2\beta_k}{r_k}\right)$$

$$g^{①}_{\mathrm{II\,I},\xi}=-\frac{\ell}{8\pi G}\Sigma\,(1-\eta_k)\,W_k\left(-(1-2\nu)\frac{\sin\beta_k}{r_k}+\frac{\sin\beta_k(2\cos^2\beta_k-1)}{r_k}\right)$$

$$g^{①}_{\mathrm{I\,II},\eta}=-\frac{\ell}{8\pi G}\Sigma\,(1-\eta_k)\,W_k\left(-(1-2\nu)\frac{\cos\beta_k}{r_k}+\frac{\cos\beta_k(2\sin^2\beta_k-1)}{r_k}\right)$$

$$g^{①}_{\mathrm{II\,II},\xi}=-\frac{\ell}{8\pi G}\Sigma\,(1-\eta_k)\,W_k\left(2(1-\nu)\frac{\cos\beta_k}{r_k}+\frac{2\cos\beta_k\sin^2\beta_k}{r_k}\right). \qquad (27)$$

Substituting Eq. (27) into Eq. (26) produces the final forms of matrix D as to node 1 in the form

$$D^{①}_{11}=-\frac{\ell}{2\pi}\sum_{k=1}^{K}\left(\cos^3\beta_k(1-\eta_k)\,W_k/r_k\right)$$

$$D^{①}_{12}=-\frac{\ell}{2\pi}\Sigma\left(\cos^2\beta_k\sin\beta_k(1-\eta_k)\,W_k/r_k\right)$$

$$D^{①}_{21}=D^{①}_{12}$$

$$D^{①}_{22}=D^{①}_{31}$$

$$D^{①}_{31}=-\frac{\ell}{2\pi}\Sigma\left(\cos\beta_k\sin^2\beta_k(1-\eta_k)\,W_k/r_k\right)$$

$$D^{①}_{32}=-\frac{\ell}{2\pi}\left(\sin^3\beta_k(1-\eta_k)\,W_k/r_k\right) \qquad (28)$$

where the replacement of function N leads to the results about node 2 as mentioned previously.

The displacements and stresses on the surface (where $p_k=0$) can be obtained as those at internal points due to the nature of fundamental solutions.

3. 4 Definition of parameters

The parameters used in the fundamental solutions and its derivatives are defined here for ascertainment.

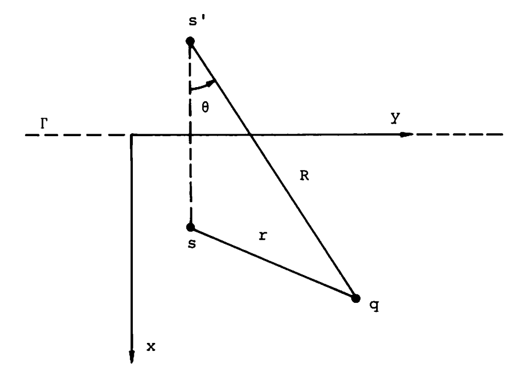

Fig. 1 load point s, its image s' and field point q which provide fundamental solutions for half-plane problem

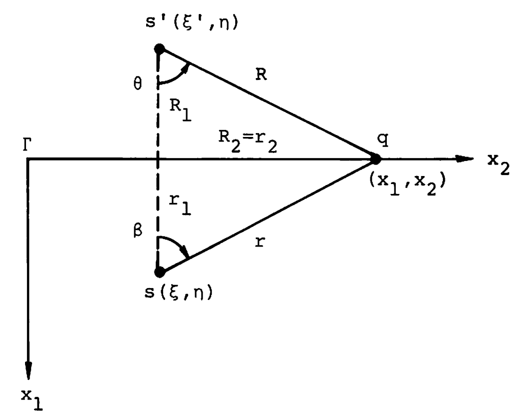

Fig. 2 Definition of θ and β when the field point q is on the surface in Melan's solution

The point s' in Fig. 1 and Fig. 2 is the image of the load point s inside the body with respect to the surface which provides complementary part of fundamental solution for half-plane problem derived from the solution due to Kelvin. Fig. 2 shows the case that the field point q is on the surface in the Melan's solution[2] (equation (6)−(8)). Following relations are apparently obtained

$$R_1 = \xi' - x_1 = \xi'$$
$$r_1 = \xi - x_1 = \xi$$
$$R_2 = r_2 = \eta - x_2$$
$$R = r \tag{29}$$

and

$$\sin\theta = \sin\beta$$
$$\cos\theta = -\cos\beta. \tag{30}$$

Derivatives of θ with respect to ξ and η are

$$\frac{\partial\theta}{\partial\xi} = \frac{\partial}{\partial\xi'}\tan^{-1}\left(\frac{\eta - x_2}{\xi' - x_1}\right)\cdot\frac{\partial\xi'}{\partial\xi} = \frac{R_2}{R^2} = \frac{\sin\theta}{R} = \frac{\sin\beta}{r}$$
$$\frac{\partial\theta}{\partial\eta} = \frac{\partial}{\partial\eta}\tan^{-1}\left(\frac{\eta - x_2}{\xi' - x_1}\right) = \frac{R_1}{R^2} = \frac{\cos\theta}{R} = -\frac{\cos\beta}{r} \tag{31}$$

which is applied to equation (26) to obtain equation (27).

For the computation of surface values, limitation form of θ leads to

$$\theta = \begin{cases} \frac{\pi}{2} & (r_2 > 0,\ r_1 = 0) \\ -\frac{\pi}{2} & (r_2 < 0,\ r_1 = 0). \end{cases} \tag{32}$$

4. Examples

Two simple and basic examples solved by linear element and half-plane implementation are presented and compared with analytical results[3].

Following calculations indicate the 'strength' of boundary element method, not only the accuracy of values obtained but the advantage in pre- and post- processing.

4. 1 Linear increasing loading perpendicular to the surface

The example consists of triangular strip loading on a finite part of semi-infinite medium under the condition of plane strain. This problem was solved by discretizing the loaded part into only two elements and three nodes (Fig. 3). Displacements and stresses at internal points are compared with analytical solutions in Table 1.

Since only relative vertical displacements have physical significance for half-plane analysis, all the vertical displacements are given relatively with reference to node 2 in Table 1. Stresses are negative on compression.

Relative displacements are shown in Fig. 4. No confining conditions are considered to calculate and figure. Such post process is very easy to perform since any desired values can be calculated directly without any interpolations in the domain. This feature is one of the great advantages of this method.

4. 2 Linear increasing loading parallel to the surface

This example is in the same way as 4.1 though loading direction is parallel to the surface (Fig. 5). Relative horizontal displacements, vertical displacements and stresses are given in Table 2 and resultant relative displacements are shown in Fig. 6. Analytical results on displacements have not been evaluated.

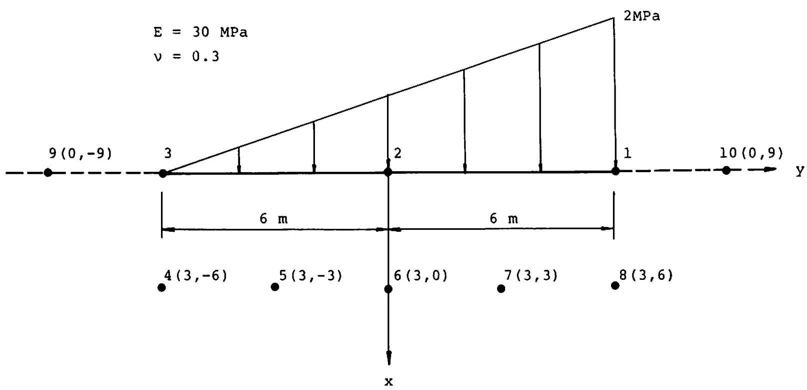

Fig. 3 Linear increasing loading perpendicular to the surface. Two boundary elements, three nodes and internal points to calculate.

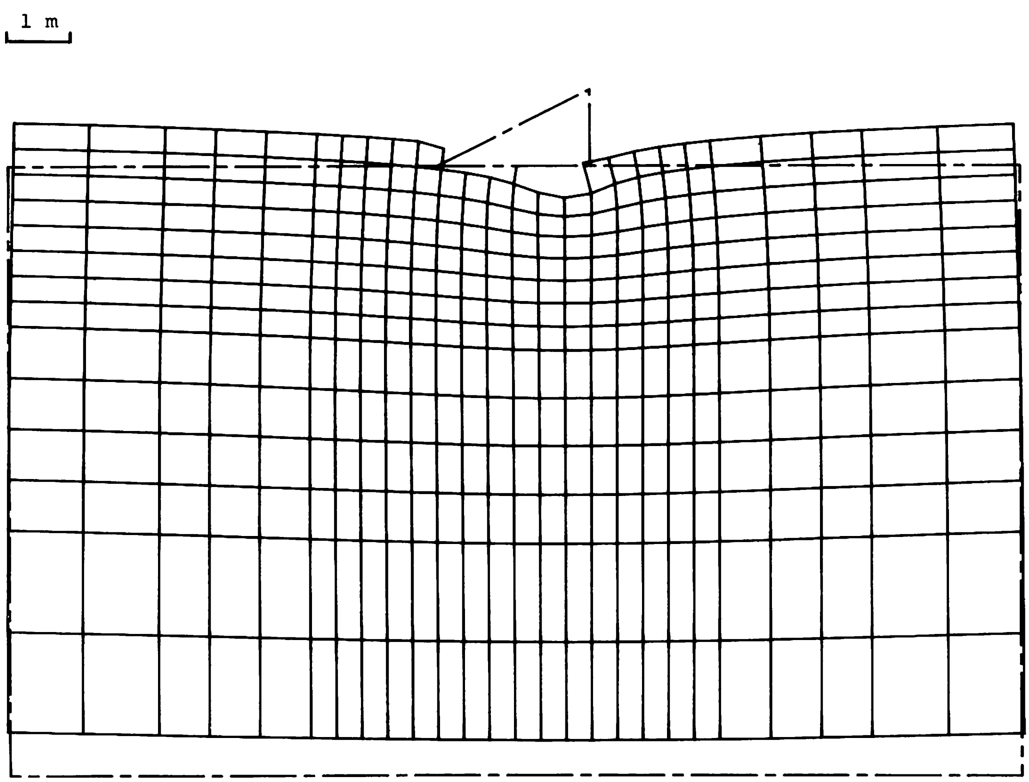

Fig. 4 Relative displacements with reference to the vertical displacement of node 2

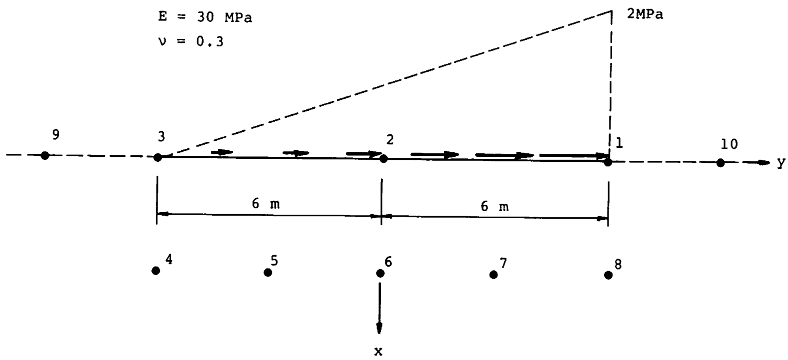

Fig. 5 Linear increasing loading parallel to the surface

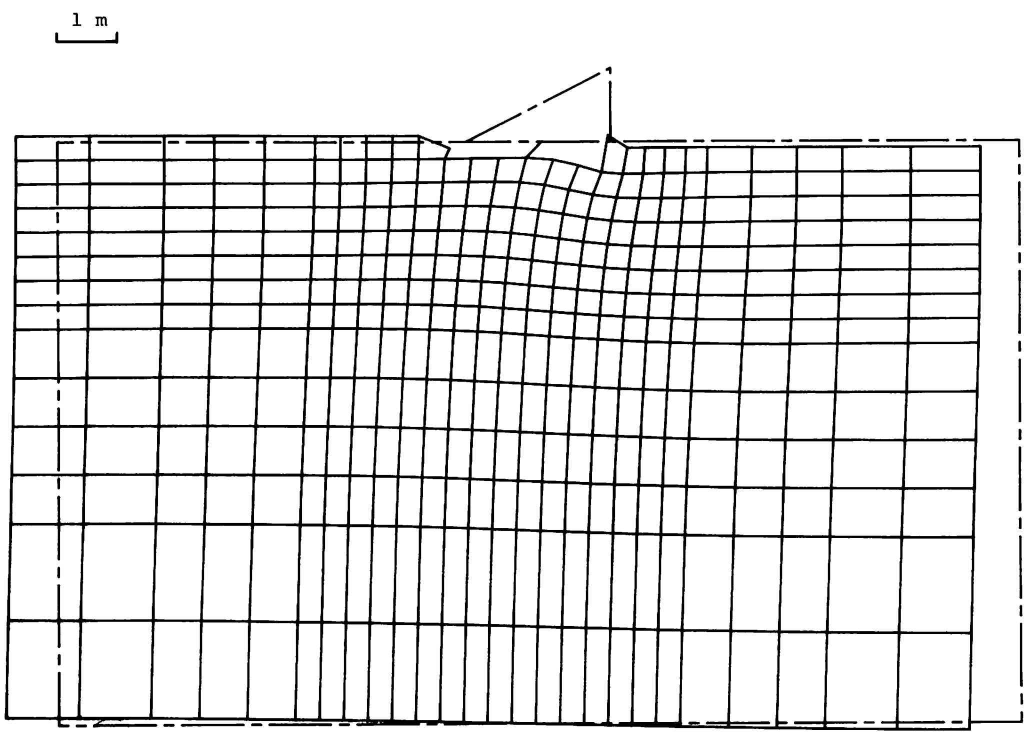

Fig. 6 Relative displacements with reference to the horizontal displacement of node 2

Table 1 Displacements and stresses by vertially increasing loading with analytical results (in parenthesis).

No.	1	2	3	4	5	6	7	8	9	10
x	0	0	0	3	3	3	3	3	0	0
y	6	0	−6	−6	−3	0	3	6	−9	9
ux-ux(2) (cm)	−4.375 (−4.476)	0.0 (0.0)	−27.574 (−27.649)	−27.371	−16.328	−6.150	−2.115	−10.346	−36.181 (−36.282)	−24.731 (−24.832)
uy (cm)	−10.4000	5.200	10.400	2.139	0.217	−0.768	−0.264	0.996	10.400	−10.400
σx (kPa)	—	—	—	−149.79 (−149.79)	−514.77 (−514.77)	−959.48 (−959.48)	−1289.7 (−1289.7)	−844.04 (−844.04)	0.0 (0.0)	0.0 (0.0)
σy (kPa)	—	—	—	−301.13 (−301.13)	−388.96 (−388.96)	−450.19 (−450.19)	−396.93 (−396.92)	−393.12 (−393.12)	0.0 (0.0)	0.0 (0.0)
σxy (kPa)	—	—	—	173.56 (173.56)	260.13 (260.13)	225.09 (225.09)	5.485 (5.482)	−425.61 (−425.61)	0.0 (0.0)	0.0 (0.0)

Table 2 Displancements and stresses by horizontally increasing loading with analytical results (in parenthesis).

No.	1	2	3	4	5	6	7	8	9	10
x	0	0	0	3	3	3	3	3	0	0
y	6	0	−6	−6	−3	0	3	6	−9	9
ux (cm)	−10.400	5.200	10.400	−9.894	−10.320	−8.973	−1.360	8.016	−7.800	7.800
uy-uy(2) (cm)	−4.375	0.0	−27.547	−26.411	−21.835	−18.112	−16.827	−17.653	−29.181	−15.024
σx (kPa)	—	—	—	173.56 (173.56)	260.13 (260.13)	225.09 (225.09)	5.4853 (5.4819)	−425.61 (−425.61)	0.0 (0.0)	0.0 (0.0)
σy (kPa)	—	—	—	677.66 (677.66)	621.67 (621.68)	343.31 (343.31)	−148.28 (−148.28)	−526.85 (−526.85)	760.94 (760.94)	−1288.26 (−1288.26)
σxy (kPa)	—	—	—	−301.13 (−301.13)	−388.96 (−388.96)	−450.19 (−450.19)	−396.93 (−396.92)	−393.12 (−393.12)	0.0 (0.0)	0.0 (0.0)

5. Conclusions

The application of boundary element method to the semi-infinite problems of 2−dimensional shows that the method has many important advantages and potentials in engineering practice.

The procedure is more accurate than discretizing the domain using finite elements because

1) elements tending to infinity are not necessary
2) interpolation approximations are not performed to obtain the domain values.

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6. References

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