On Dynamic Pressure to Side Wall of a Box Filled Sand

by
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ABSTRACT

Plain strain state equilibrium is taken for the above subject and analysis will be carried on by means of finite Fourier transforms.

1. General Solution

Under the stationary vibration $\sin \gamma t$, we can write the displacement Components in the x, y directions as $U\sin \gamma t$, and the component of stresses as $\sigma_x \sin \gamma t$, $\sigma_y \sin \gamma t$, $\tau_{xy} \sin \gamma t$. (Fig. 1)

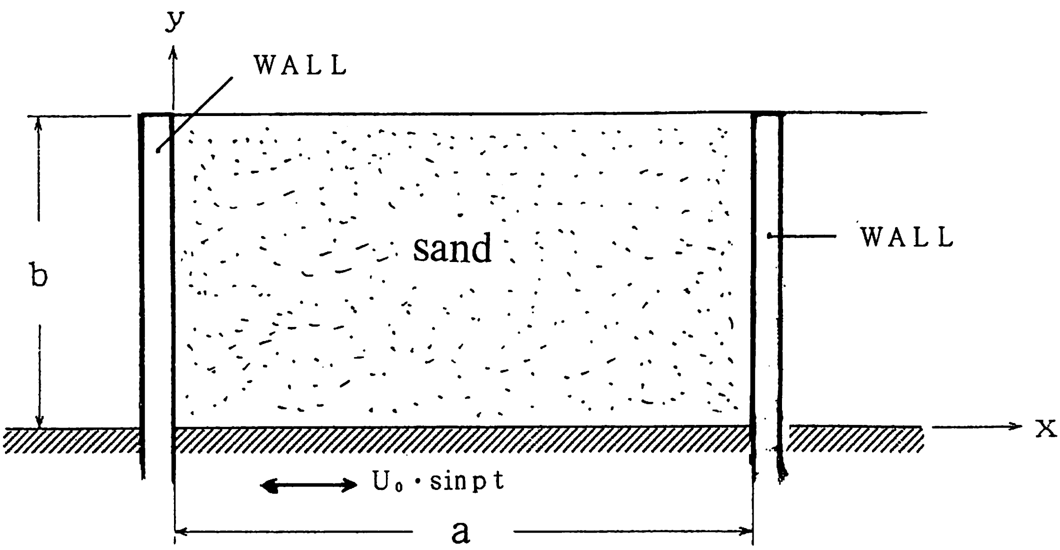

Fig. 1 Model of Analysis

So doing, the equilibrium of forces are expressed

by $$\frac{\partial \sigma_x}{\partial x}+\frac{\partial \tau_{xy}}{\partial y}+\rho \ddot{U}=-\rho \ddot{U}_0 \qquad (1)$$

$$\frac{\partial \sigma_y}{\partial y}+\frac{\partial \tau_{xy}}{\partial_x}+\rho \ddot{V}=-g\rho \qquad (2)$$

in which U is the displacement in x direction

V is the displacement in y direction with ρ q

Hooke's Law is expressed by

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$$\rho\left|\begin{array}{cc} c_1^2\dfrac{\partial}{\partial x} & (c_1^2-2c_2^2)\dfrac{\partial}{\partial y} \\ (c_1^2-2c_2^2)\dfrac{\partial}{\partial x} & c_2^1\dfrac{\partial}{\partial y} \\ c_2^2\dfrac{\partial}{\partial y} & c_2^2\dfrac{\partial}{\partial x} \end{array}\right| \left\{\begin{array}{c} U \\ V \end{array}\right\} = \left\{\begin{array}{c} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{array}\right\} \qquad (3)$$

where $(2G+\lambda)=C_1^2\rho$ $\quad G=C_2^2\rho$

ρ = density of Sand $\quad C_1 = C_2$ = bulk and distorsion velocities

Multiplying L_1, L_2 with (1) and (2) and integrating by parts, We have

$$\int_0^b \frac{\sigma_x}{\rho}L_1\Big]_0^a dy - C_1^2\int_0^b U\frac{\partial L_1}{\partial x}\Big]_0^a dy - (C_1^2-2C_2^2)\int_0^a V\frac{\partial L_1}{\partial x}\Big]_0^b dx$$
$$+\int_0^a \frac{\tau_{xy}}{\rho}L_1\Big]_0^b dx - C_2^2\int_0^a U\frac{\partial L_1}{\partial y}\Big]_0^b dx - C_2^2\int_0^b V\frac{\partial L_1}{\partial y}\Big]_0^a dy$$
$$+\int_A U\left(C_1^2\frac{\partial^2 L_1}{\partial x^2}+C_2^2\frac{\partial^2 L_1}{\partial y^2}+\gamma^2\right)dA+\int_A V(C_1^2-C_2^2)\frac{\partial^2 L_1}{\partial x\partial y}dA=-\int_A U_0L_1dA \qquad (4)$$

$$\int_0^a \frac{\sigma_y}{\rho}L_2\Big]_0^b dx - C_1^2\int_0^a V\frac{\partial L_2}{\partial y}\Big]_0^b dx$$
$$-(C_1^2-2C_2^2)\int_0^b U\frac{\partial L_2}{\partial y}\Big]_0^a dy + \int_0^b \frac{\tau_{xy}}{\rho}L_2\Big]_0^a dy - C_2^2\int_0^b V\frac{\partial L_2}{\partial x}\Big]_0^a dy$$
$$-C_2^2\int_0^a U\frac{\partial L_2}{\partial x}\Big]_0^b dx + \int_A V\left(C_1^2\frac{\partial^2 L_2}{\partial y^2}+C_2^2\frac{\partial^2 L_2}{\partial x^2}+\gamma^2\right)dA$$
$$+\int_A U(C_1^2-C_2^2)\frac{\partial^2 L_2}{\partial x\partial y}dA=0 \qquad (5)$$

Let $\quad L_1 = \sin Mx \cdot \sin Ny$

where $\quad M=\dfrac{m\pi}{a},\ N=\dfrac{n\pi}{b};\ m, n = 1.\ 2.\ 3.\ \cdots\cdots$

$$-C_1^2\{S_n[U_{ay}](-1)^m - S_n[U_{0y}]\}M - C_2^2\{S_m[U_{xb}](-1)^n - S_m[U_{x0}]\}N$$
$$-S_mS_n[U]\cdot(C_1^2M^2+C_2^2N^2-\gamma^2)+C_mC_n[V]\cdot(C_1^2-C_2^2)MN$$
$$=-\gamma^2S_mS_n[U_0] \qquad (6)$$

Let $\quad L_2 = \cos Mx \cdot \cos Ny$

$$\frac{1}{\rho}\{C_m[(\sigma_y)_{y=b}](-1)^n - C_m[(\sigma_y)_{y=0}]\}+(C_1^2-2C_2^2)N\{S_n[U_{ay}](-1)^m - S_n[U_{0y}]\}$$
$$+\frac{1}{\rho}\{C_n[(\tau_{xy})_{y=a}](-1)^m - C_n[(\tau_{xy})_{x=0}]+C_2^2M\{S_m[U_{xb}](-1)^n - S_m[U_{x0}]\}$$
$$-C_mC_n[V](C_1^2N^2+C_2^2M^2-\gamma^2)+S_mS_n[U](C_1^2-C_2^2)MN=0 \qquad (7)$$

The case of $m=0$, $n \neq 0$, (7) is expressed as

$$\frac{1}{\rho}\{C_0[(\sigma_y)_{y=b}](-1)^n - C_0[(\sigma_y)_{y=0}\}+(C_1^2-2C_2^2)N\{S_n[U_{ay}]-S_n[U_{0y}]\}$$
$$\frac{1}{\rho}C_n[(\tau_{xy})_{x=0} - C_n[(\tau_{xy})_{x=0}\}-C_0C_n[V](C_1^2N^2-\gamma^2)=0 \qquad (8)$$

The case of $m \neq 0$, $n = 0$, (7) is also expressed as

$$\frac{1}{\rho}\{ C_m [(\sigma_y)_{y=b}] - C_m [(\sigma_y)_{y=0}]\} + \frac{1}{\rho}\{ C_0 [(\tau_{xy})_{x=0}] - C_0 [(\tau_{xy})_{x=0}]\}$$
$$+ C_2^2 M \{ S_m [U_{xb}] - S_m [U_{x0}]\} - C_m C_0 [V](C_2^2 M^2 - \gamma^2) = 0 \tag{9}$$

Then, the case of $m = 0$, $n = 0$, (7) is shown as following

$$\frac{1}{\rho}\{ C_0 [(\sigma_y)_{y=b}] - C_0 [(\sigma_y)_{y=0}]\} + \frac{1}{\rho}\{ C_0 [(\tau_{xy})_{x=a}] - C_0 [(\tau_{xy})_{x=0}]\}$$
$$+ C_0 C_0 [V] \gamma^2 = 0 \tag{10}$$

For convenience sake, putting

$$S_m S_n [U] = U_{mn}, \qquad C_m C_n [V] = V_{mn},$$

then we have

$$\begin{vmatrix} C_1^2 M^2 + C_2^2 N^2 - \gamma^2 & -[C_1^2 - C_2^2) MN \\ -(C_1^2 - C_2^2) MN & C_1^2 N^2 + C_2^2 M^2 - \gamma^2 \end{vmatrix} \begin{Bmatrix} U_{mn} \\ V_{mn} \end{Bmatrix}$$
$$= \begin{Bmatrix} \gamma^2 S_m S_n [U_0] - N(-1)^n C_2^2 S_m [U_{xb}] \\ -\frac{1}{\rho} C_m [(\sigma_y)_{y=0}] + M(-1)^n C_2^2 S_m [U_{xb}] \\ +\frac{1}{\rho}\{ C_n [(\tau_{xy})_{x=a}](-1)^m - C_n [(\tau_{xy})_{x=0}]\} \end{Bmatrix} \tag{11}$$

By putting again in the above

$$\frac{C_2^2}{C_1^2} = \kappa, \quad \frac{\gamma^2}{C_1^2} = p^2, \quad F_{mn} = p^2 S_m S_n [U_0] = U_0 p^2 (1-(-1)^m)(1-(-1)^n) MN$$
$$S_m [U_{xb}] = A_m, \quad \frac{1}{\rho} C_m [(\sigma_y)_{y=0}] = B_m$$

And, the boundary condition are so given as to come
for $x = 0, a$, $u = 0$, $\tau_{xy} = 0$; and $y = 0$, $\sigma_y = 0$, $\tau_{xy} = 0$; and
$y = b$, $v = 0$, $u = 0$.

$$\begin{vmatrix} M^2 + \kappa N^2 - p^2 & -(1-\kappa) MN \\ -(1-\kappa) MN & \kappa M^2 + N^2 - p^2 \end{vmatrix} \begin{Bmatrix} U_{mn} \\ V_{mn} \end{Bmatrix} = \begin{Bmatrix} F_{mn} - \kappa A_m (-1)^n N \\ -B_m + \kappa A_m (-1)^m M \end{Bmatrix} \tag{12}$$

Then,

$$U_{mn} = [F_{mn}(\kappa M^2 + N^2 - p^2) + \kappa A_m (-1)^n N \{(1-2\kappa) M^2 - N^2 + p^2\}$$
$$-(1-\kappa) MNB_m] / \Omega_{mn} \tag{13}$$

$$V_{mn} = [F_{mn}(1-\kappa) MN + \kappa A_m M \{ M^2 - (1-2\kappa) N^2 - p^2\}$$
$$-(M^2 + \kappa N^2 - p^2) B_m] / \Omega_{mn} \tag{14}$$

where

$$\Omega_{mn} = \kappa (M^2 + N^2 - p^2)(M^2 + N^2 - \frac{p^2}{\kappa}) = \kappa HH'$$
$$H = M^2 + N^2 - p^2$$
$$H' = M^2 + N^2 - \frac{p^2}{\kappa}$$

These result are summed up to be following solutions by the finite Fourier inversion formula.

$$U=\sum_m \frac{(1-(-1)^m)}{2}\sin Mx\left[\frac{4U_0}{b}\frac{b}{m\pi}\left\{(1-Q_m(\eta)\frac{p^2}{\beta_m{}^2}\right.\right.$$
$$\left.+(Q'_m(\eta)-Q_m(\eta))\right\}-\frac{2hA_m e\pi}{b^2p^2}\frac{b}{\pi}\{2(me)^2(R'_m(\eta)$$
$$-R_m(\eta))-p'^2R'_m(\eta)\}+\frac{2B_m e}{bp^2}\frac{b}{\pi}(me)(R_m(1-\eta)$$
$$\left.-R'_m(1-\eta))\right] \tag{15}$$

$$V=\sum_m \frac{(1-(-1)^m)}{2}\cos Mx\left[\frac{4U_0}{b}\frac{be}{\pi}\left(\frac{\psi'_m(\eta)}{\beta'_m}-\frac{\psi_m(\eta)}{\beta_m}\right)\right.$$
$$+\frac{2hA_m e\pi}{b^2p^2}\frac{b}{\pi}(me)\left\{2(\beta'_m\phi'_m(\eta)-\beta_m\phi_m(\eta))+\frac{p'^2}{\beta'_m}\phi'_m(\eta)\right\}$$
$$\left.-\frac{2B_m e}{bp^2}\frac{b}{\pi}\left\{\frac{(me)^2}{\beta'_m}\phi'_m(1-\eta)-\beta_m\phi_m(1-\eta)\right\}\right] \tag{16}$$

where

$$Q_m(\eta)=\frac{ch\left(\beta_m\pi\left(\frac{1}{2}-\eta\right)\right)}{ch(\beta_m\pi/2)}\qquad\left[=\frac{\cos\left(\beta_m\pi\left(\frac{1}{2}-\eta\right)\right)}{\cos(\beta_m\pi/2)}\right]$$

$$R_m(\eta)=\frac{sh(\beta_m\pi\eta)}{sh(\beta_m\pi)}\qquad\left[=\frac{\sin(\beta_m\pi\eta)}{\sin(\beta_m\pi)}\right]$$

$$R_m(1-\eta)=\frac{sh(\beta_m\pi(1-\eta))}{sh(\beta_m\pi)}\qquad\left[=\frac{\sin(\beta_m\pi(1-\eta))}{\sin(\beta_m\pi)}\right]$$

$$\phi_m(\eta)=\frac{ch(\beta_m\pi\eta)}{sh(\beta_m\pi)}\qquad\left[=\frac{\cos(\beta_m\pi\eta)}{\sin(\beta_m\pi)}\right]$$

$$\phi_m(1-\eta)=\frac{ch(\beta_m\pi(1-\eta))}{sh(\beta_m\pi)}\qquad\left[=\frac{\cos(\beta_m\pi(1-\eta))}{\sin(\beta_m\pi)}\right]$$

$$\psi_m(\eta)=\frac{sh\left(\beta_m\pi\left(\frac{1}{2}-\eta\right)\right)}{ch(\beta_m\pi/2)}\qquad\left[=\frac{\sin\left(\beta_m\pi\left(\frac{1}{2}-\eta\right)\right)}{\cos(\beta_m\pi/2)}\right]$$

$$Q'_m(\eta)=\frac{ch\left(\beta'_m\pi\left(\frac{1}{2}-\eta\right)\right)}{ch(\beta'_m\pi/2)}\qquad\left[=\frac{\cos\left(\beta'_m\pi\left(\frac{1}{2}-\eta\right)\right)}{\cos(\beta'_m\pi/2)}\right]$$

$$R'_m(\eta)=\frac{sh(\beta'_m\pi\eta)}{sh(\beta'_m\pi)}\qquad\left[=\frac{\sin(\beta'_m\pi\eta)}{\sin(\beta'_m\pi)}\right]$$

$$R'_m(1-\eta)=\frac{sh(\beta'_m\pi(1-\eta))}{sh(\beta'_m\pi)}\qquad\left[=\frac{\sin(\beta'_m\pi(1-\eta))}{\sin(\beta'_m\pi)}\right]$$

$$\phi'_m(\eta) = \frac{ch(\beta'_m \pi \eta)}{sh(\beta'_m \pi)} \qquad \left[= \frac{\cos(\beta'_m \pi \eta)}{\sin(\beta'_m \pi)} \right]$$

$$\phi'_m(1-\eta) = \frac{ch(\beta'_m \pi(1-\eta))}{sh(\beta'_m \pi)} \qquad \left[= \frac{\cos(\beta'_m \pi(1-\eta))}{\sin(\beta'_m \pi)} \right]$$

$$\psi'_m(\eta) = \frac{sh\left(\beta'_m \pi\left(\frac{1}{2}-\eta\right)\right)}{ch(\beta'_m \pi/2)} \qquad \left[= \frac{\sin\left(\beta'_m \pi\left(\frac{1}{2}-\eta\right)\right)}{\cos(\beta'_m \pi/2)} \right]$$

$$\beta_m^2 = (me)^2 - p^2 \qquad M = \frac{m\pi}{a} \qquad \left[\text{cage of imaginary} \right]$$

$$\beta_m'^2 = (me)^2 - p'^2 \qquad \eta = \frac{y}{b}$$

$$p'^2 = p^2/h \qquad e = \frac{b}{a}$$

$$h = \frac{C_2^2}{C_1^2} = \frac{G}{2G+\lambda} = \frac{1-2\nu}{2(1-\nu)}$$

ν : poisson's ratio

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