振動台上の砂箱内地盤モデルにおける動的応力と変位の弾性解

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Two Dimentional Elastic Solution of Dynamic Stress and Displacement for the Sand Ground Model on the Shaking Table

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要旨

振動時地盤における動的挙動の理論解析において，振動台上に設置の砂箱内に作成された地盤モデル（写真１及び図１）における動的変位と応力を平面歪状態として動的２次元弾性解を有限フーリエ変換により求めた詳細を報告するものである。

Abstract

Under the stationary vibration on the shaking table, in which plain strain state equilibrium is taken for the above subject, we can write the dynamic displacement components and the component of stresses by means of Finite Fourier Transforms.

1．はじめに

地震等，振動時の地盤における動的土圧（慣性力）の分布形態は常時と異なると考えられるが，現行の設計では震度法に根拠を置いて擬静的に取り扱っている。そこで本稿では振動台に載せたアクリル製砂箱の中にモデル地盤を設置した場的の動的２次元弾性解の詳細を報告するものである。

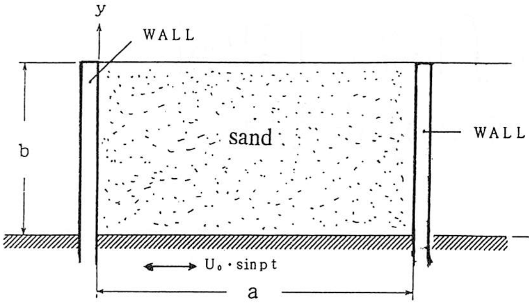
尚，供試体材料は標準砂であり，均質・等方性とし，平面ひずみ状態と仮定する。

2．理論解析

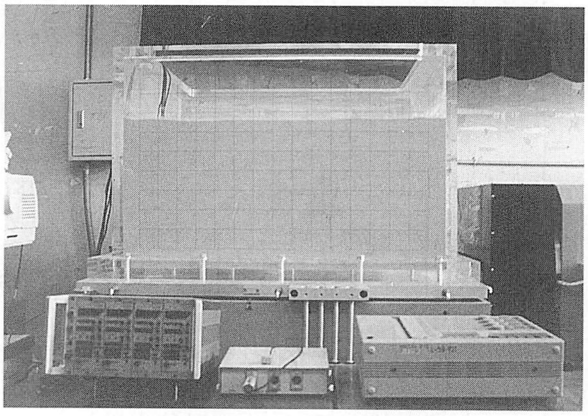
基礎方程式は、二次元弾性論より釣り合い式が、次のように示される。

$$\frac{\partial \sigma_x}{\partial x}+\frac{\partial \tau_{x,y}}{\partial y}+\rho\ddot{u}=\rho\ddot{u}_0 \qquad (1)$$

$$\frac{\partial \tau_{x,y}}{\partial x}+\frac{\partial \sigma_y}{\partial y}+\rho\ddot{v}=-g\rho \qquad (2)$$



図－１　解析モデル図



写真－１　解析地盤モデル

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u,vは，各々，x方向，y方向の相対変位で，u_0は振動台からの入力振動であり，

$\ddot{u}=\frac{\partial^2 u}{\partial t^2}$，$\ddot{v}=\frac{\partial^2 v}{\partial t^2}$，$\ddot{u}_0=\frac{\partial^2 u_0}{\partial t^2}$ である。

又，$\frac{(2G+\lambda)}{\rho}=C_1^2$，$\frac{G}{P}=C_2^2$ であり，ρは砂の密度，C_1は弾性波の速度，C_2は剪断速度となる。

又，Hookeの法則より，

$$\sigma_x=\rho C_1^2\frac{\partial u}{\partial x}+\rho(C_1^2-2C_1^2)\frac{\partial v}{\partial y} \qquad (3)$$

$$\sigma_y=\rho(C_1^2-2C_2^2)\frac{\partial u}{\partial x}+\rho C_1^2\frac{\partial v}{\partial y} \qquad (4)$$

$$\tau_{x,y}=\rho C_2^2\frac{\partial u}{\partial_y}+\rho C_2^2\frac{\partial_v}{\partial_x} \qquad (5)$$

(1)，(2)式をフーリエ定積変換すると，

$$\int_0^a\int_0^b\frac{1}{\rho}\left\{\frac{\partial\sigma_x}{\partial_x}+\frac{\partial\tau_{x,y}}{\partial_y}+\rho\ddot{u}=-\rho\ddot{u}_0\right\}L_1dydx \qquad (6)$$

$$\int_0^b\int_0^a\frac{1}{\rho}\left\{\frac{\partial\tau_{x,y}}{\partial_x}+\frac{\partial\sigma_y}{\partial_y}+\rho\ddot{v}=-g\rho\right\}L_2dxdy \qquad (7)$$

ここで，$L_1=\sin(Mx)\cdot\sin(Ny)$，$L_2=\cos(Mx)\cdot\cos(Ny)$であり，$M=\frac{m\pi}{a}$，$N=\frac{n\pi}{b}$;m,n=1,2,3,　である。

(6)式の第1項について計算すると，

$$\int_0^b\int_0^a\frac{1}{\rho}\frac{\partial\sigma_x}{\partial x}L_1\,dx\,dy$$

$$=\frac{1}{\rho}\int_0^b\int_0^a(\sigma_x)'_{,}L_1\,dx\,dy$$

$$=\frac{1}{\rho}\int_0^b\left\{[\sigma_xL_1]_0^a-\int_0^a\sigma_x\frac{\partial L_1}{\partial x}\,dx\right\}dy$$

ここで，Hookeの法則より，

$$\sigma_x=\rho\,C_1^2\frac{\partial u}{\partial x}+\rho(C_1^2-C_2^2)\frac{\partial v}{\partial y}$$

を代入すると，

$$=\frac{1}{\rho}\int_0^b\left\{[\sigma_xL_1]_0^a-\int_0^a\left(\rho\,C_1^2\frac{\partial u}{\partial x}+\rho(C_1^2-C_2^2)\frac{\partial v}{\partial y}\right)\frac{\partial L_1}{\partial x}\,dx\right\}dy$$

$$=\frac{1}{\rho}\int_0^b[\sigma_xL_1]_0^a dy-\int_0^b\int_0^a C_1^2\frac{\partial u}{\partial x}\frac{\partial L_1}{\partial x}\,dx\,dy-\int_0^a\int_0^b(C_1^2-C_2^2)\frac{\partial v}{\partial y}\frac{\partial L_1}{\partial x}\,dy\,dx$$

$$=\frac{1}{\rho}\int_0^b[\sigma_xL_1]_0^a dy-C_1^2\int_0^b\left\{\left[u\frac{\partial L_1}{\partial x}\right]_0^a-\int_0^a u\frac{\partial^2L_1}{\partial x^2}\,dx\right\}dy-(C_1^2-2C_2^2)\int_0^a\left\{\left[v\frac{\partial L_1}{\partial x}\right]_0^b-\int_0^b\frac{\partial^2L_1}{\partial xy}\,dx\right\}$$

$$=\int_0^b\left[\frac{\sigma_x}{\rho}L_1\right]_0^a dy-C_1^2\int_0^b\left[u\frac{\partial L_1}{\partial x}\right]_0^a dy+C_1^2\int_0^b\int_0^a u\frac{\partial^2L_1}{\partial x^2}\,dx\,dy-(C_1^2-2C_2^2)\int_0^a\left\{\left[v\frac{\partial L_1}{\partial x}\right]_0^b dx\right.$$

$$+(C_1^2-2C_2^2)\int_0^a\int_0^b v\frac{\partial^2L_1}{\partial x\partial y}\,dy\,dx \qquad (6.1)$$

同様に，(6)式の第2項について計算すると，

$$\int_0^a\int_0^b\frac{1}{\rho}\frac{\partial\tau_{x,y}}{\partial y}L_1dy\,dx$$

$$=\frac{1}{\rho}\int_0^a\int_0^b(\tau_{x,y})'L_1dy\,dx=\frac{1}{\rho}\int_0^a\left\{[\tau_{x,y}L_1]_0^b-\int_0^b\tau_{x,y}\frac{\partial L_1}{\partial y}\,dy\right\}dx$$

ここで，Hookeの法則より

$$\tau_{x,y}=\rho\, C_1^2\frac{\partial u}{\partial y}+\rho\, C_2^2\frac{\partial v}{\partial x}$$

を代入すると，

$$=\frac{1}{\rho}\int_0^a\left\{[\tau_{x,y}L_1]_0^b-\int_0^b\left(\rho\, C_2^2\frac{\partial u}{\partial y}+\rho\, C_2^2\frac{\partial v}{\partial x}\right)\frac{\partial L_1}{\partial y}\,dy\right\}dx$$

$$=\int_0^a\left[\frac{\tau_{x,y}}{\rho}L_1\right]_0^b dx-\int_0^a\int_0^b C_2^2\frac{\partial u}{\partial y}\frac{\partial L_1}{\partial y}\,dy\,dx-\int_0^b\int_0^a C_2^2\frac{\partial v}{\partial x}\frac{\partial L_1}{\partial y}\,dx\,dy$$

$$=\int_0^a\left[\frac{\tau_{x,y}}{\rho}L_1\right]_0^b dx-C_2^2\int_0^a\left\{\left[u\frac{\partial L_1}{\partial y}\right]_0^b-\int_0^a u\frac{\partial^2 L_1}{\partial y^2}\,dy\right\}dx-C_2^2\int_0^b\left\{\left[v\frac{\partial L_1}{\partial y}\right]_0^a-\int_0^a v\frac{\partial^2 L_1}{\partial x\partial y}\,dx\right\}dy$$

$$=\int_0^a\left[\frac{\tau_{x,y}}{\rho}L_1\right]_0^b dx-C_2^2\int_0^a\left[u\frac{\partial L_1}{\partial y}\right]_0^b dx+C_2^2\int_0^a\int_0^b u\frac{\partial^2 L_1}{\partial y^2}\,dy\,dx-C_2^2\int_0^b\left[v\frac{\partial L_1}{\partial y}\right]_0^a dy$$

$$+C_2^2\int_0^b\int_0^a v\frac{\partial^2 L_1}{\partial x\partial y}\,dx\,dy \qquad (6.2)$$

同様に，(6)式の第３項について計算すると，

$$\int_0^b\int_0^a\frac{1}{\rho}\rho\,\ddot{u}\,L_1\,dx\,dy$$

$$u=u\,\sin(\gamma t)$$

$$\frac{\partial u}{\partial t}=\gamma\, u\cos(\gamma t)$$

$$\therefore\ \int_0^b\int_0^a\{-\gamma^2 u\sin(\gamma t)\}L_1dxdy$$

$$=-\gamma^2\int_A^0 u\sin(\gamma t)L_1dA\ =-\gamma^2\int_A^0 u(\gamma t)L_1dA \qquad (6.3)$$

同様に，(6)の右辺について計算すると，

$$\int_0^b\int_0^a-\frac{1}{\rho}\rho\,\ddot{u}_0L_1\,dx\,dy$$

$u_0=u_0\sin(\gamma t)$　とおく

$$\frac{\partial u_0}{\partial t}=\gamma\, u_0\cos(\gamma t)$$

$$\frac{\partial^2 u_0}{\partial t^2}=-\gamma^2 u_0\sin(\gamma t)$$

$$\therefore\gamma^2\int_A^0 u_0L_1dA \qquad (6.4)$$

(6.1)，(6.2)，(6.3)，(6.4)式を合わせると，

$$\therefore\int_0^b\left[\frac{\sigma_x}{\rho}L_1\right]_0^a dy-C_1^2\int_0^b\left[u\frac{\partial L_1}{\partial x}\right]_0^a dy-(C_1^2-2C_2^2)\int_0^a\left[v\frac{\partial L_1}{\partial x}\right]_0^b dx+\int_0^a\left[\frac{\tau_{x,y}}{\rho}L_1\right]_0^b dx-C_2^2\int_0^a\left[u\frac{\partial L_1}{\partial y}\right]_0^b dx$$

$$-C_2^2\int_0^b\left[v\frac{\partial L_1}{\partial x}\right]_0^b dy+\int_A^0 u\left(C_1^2\frac{\partial^2 L_1}{\partial x^2}+C_2^2\frac{\partial^2 L_1}{\partial y^2}-\gamma^2L_1\right)dA+\int_A^0 v(C_1^2-C_2^2)\frac{\partial^2 L_1}{\partial x\partial y}\,dA=\gamma^2\int_A^0 u_0L_1dA \qquad (6.5)$$

次に，(6.5)式について

$L_1=\sin(Mx)\sin(Ny)$　と置いて，

$M=\frac{m\pi}{a},\ N=\frac{n\pi}{b}\quad : m,n=1,2,3$

とした時，

第１項から順次，第10項まで以下の様に示される。

$$\int_0^b\left[\frac{\sigma_x}{\rho}L_1\right]_0^a$$

$$=\int_0^b\left[\frac{\sigma_x}{\rho}\sin\left(\frac{m\pi}{a}x\right)\sin(Ny)\right]_0^a dy=0 \qquad (6.5.1)$$

第２項

$$-C_1^2\int_0^b\left[u\frac{\partial L_1}{\partial x}\right]_0^a dy$$

$$=-C_1^2\int_0^b\left[u\ M\cos(Mx)\sin(Ny)\right]_0^a dy$$

$$=-C_1^2\int_0^b\left[u\ M\cos(Mx)\sin(Ny)\right]_0^a-M\sin(Ny)dy$$

$$=-C_1^2\int_0^b\{u_{ay}\cos(m\pi)-u_{0y}(1)\}M\sin(Ny)dy$$

$$=-C_1^2\{S_n[u_{ay}](-1)^m-S_n[u_{0y}]\}M \qquad (6.5.2)$$

第３項

$$-(C_1^2-2C_2^2)\int_0^a\left[v\frac{\partial L_1}{\partial x}\right]_0^b dx$$

$$=-(C_1^2-2C_2^2)\int_0^b\left[v\cos(Mx)\sin\left(\frac{n\pi}{b}y\right)\right]_0^b dx=0 \qquad (6.5.3)$$

第４項

$$\int_0^a\left[\frac{\tau_{x,y}}{\rho}L_1\right]_0^b dx$$

$$=\int_0^a\left[\frac{\tau_{x,y}}{\rho}\sin(Mx)\sin\left(\frac{n\pi}{b}y\right)\right]_0^b dx=0 \qquad (6.5.4)$$

第５項

$$-C_2^2\int_0^a\left[u\frac{\partial L_1}{\partial y}\right]_0^b dx$$

$$=-C_2^2\int_0^a\left[u\sin(Mx)N\cos(Ny)\right]_0^b dx$$

$$=-C_2^2\int_0^a\left[u\cos(Ny)\right]_0^b N\sin(Mx)dx$$

$$=-C_2^2\int_0^b\{u_{x,b}\cos(n\pi)-u_{x,0}(1)\}N\sin(Mx)dx$$

$$=-C_2^2\{S_m[u_{x,b}](-1)^n-S_m[u_{x,0}]\}N \qquad (6.5.5)$$

第6項

$$-C_2^2\int_0^b\left[v\frac{\partial L_1}{\partial y}\right]_0^a dy$$

$$=-C_2^2\int_0^b\left[v\sin\left(\frac{m\pi}{a}x\right)N\cos(Ny)\right]_0^a dy=0 \qquad (6.5.6)$$

第7項

$$\int_A^0 u\, C_1^2\frac{\partial^2 L_1}{\partial x^2}dA$$

$$=-C_1^2\int_A^0 u\, M^2\sin(Mx)\sin(Ny)dA$$

$$=-C_1^2 M^2 S_mS_n[u] \qquad (6.5.7)$$

第8項

$$\int_A^0 u\, C_2^2\frac{\partial^2 L_1}{\partial y^2}dA$$

$$=-C_2^2\int_A^0 u\, N^2\sin(Mx)\sin(Ny)dA$$

$$=-C_2^2 N^2 S_mS_n[u] \qquad (6.5.8)$$

第9項

$$-\int_A^0 u\gamma^2 L_1 dA$$

$$=-\gamma^2\int_A^0 u\sin(Mx)\sin(Ny)dA$$

$$=-\gamma^2 S_mS_n[u] \qquad (6.5.9)$$

第10項

$$\int_A^0 v(C_1^2-C_2^2)\frac{\partial^2 L_1}{\partial x\partial y}dA$$

$$=(C_1^2-C_2^2)\int_A^0[v]M\cos(Mx)N\cos(Ny)dA$$

$$=(C_1^2-C_2^2)C_mC_n[v]MN \qquad (6.5.10)$$

次に右辺の項は次の様に示される。

$$\gamma^2\int_A^0 u_0 L_1 dA$$

$$=\gamma^2\int_A^0 u_0\sin(Mx)\sin(Ny)dA=\gamma^2S_mS_n[u_0] \qquad (6.5.11)$$

(6.5.1)から(6.5.11)をまとめると,

$$-C_1^2\{S_n[u_{a,y}](-1)^m-\{S_n[u_{0,y}]\}M-C_2^2\{S_m[u_{x,b}](-1)^n-\{S_m[u_{x,0}]\}N$$
$$-S_mS_n[u](C_1^2M^2+C_2^2N^2+\gamma^2)=C_mC_n[v](C_1^2-C_2^2)MN=\gamma^2S_mS_n[u_0] \qquad (6.6)$$

(7)式の第1項について計算すると，

$$\int_0^b\int_0^a \frac{1}{\rho}\frac{\partial \tau_{x,y}}{\partial x} L_2\,dx\,dy$$

$$=\frac{1}{\rho}\int_0^b\int_0^a (\tau_{x,y})' L_2\,dx\,dy$$

$$=\frac{1}{\rho}\int_0^b\left\{\left[\tau_{x,y} L_2\right]_0^a-\int_0^a \tau_{x,y}\frac{\partial L_2}{\partial x}dx\right\}dy$$

ここで，Hookeの法則より

$$\tau_{x,y}=\rho\,C_2^2\frac{\partial u}{\partial y}+\rho\,C_2^2\frac{\partial v}{\partial x}$$

を代入すると，

$$=\frac{1}{\rho}\int_0^b\left\{\left[\tau_{x,y} L_2\right]_0^a-\int_0^a\left(\rho\,C_2^2\frac{\partial u}{\partial y}+\rho\,C_2^2\frac{\partial v}{\partial x}\right)\frac{\partial L_2}{\partial x}dx\right\}dy$$

$$=\int_0^b\left[\frac{\tau_{x,y}}{\rho}L_2\right]_0^a dy-\int_0^a\int_0^b C_2^2\frac{\partial u}{\partial y}\frac{\partial L_2}{\partial x}dy\,dx-\int_0^b\int_0^a C_2^2\frac{\partial v}{\partial x}\frac{\partial L_2}{\partial x}dx\,dy$$

$$=\int_0^b\left[\frac{\tau_{x,y}}{\rho}L_2\right]_0^a dy-\int_0^a\int_0^b C_2^2\frac{\partial u}{\partial y}\frac{\partial L_2}{\partial x}dy\,dx-\int_0^b\int_0^a C_2^2\frac{\partial v}{\partial x}\frac{\partial L_2}{\partial x}dx\,dy$$

$$=\int_0^b\left[\frac{\tau_{x,y}}{\rho}L_2\right]_0^a dy-C_2^2\int_0^a\left\{\left[u\frac{\partial L_2}{\partial x}L_2\right]_0^b-\int_0^b u\frac{\partial^2 L_2}{\partial x\partial y}dy\right\}dx-C_2^2\int_0^b\left\{\left[v\frac{\partial^2 L_2}{\partial x}\right]_0^a-\int_0^a v\frac{\partial^2 L_2}{\partial x^2}dx\right\}dy$$

$$=\int_0^b\left[\frac{\tau_{x,y}}{\rho}L_2\right]_0^a dy-C_2^2\int_0^a\left[u\frac{\partial L_2}{\partial x}L_2\right]_0^b dx+C_2^2\int_0^a\int_0^b u\frac{\partial^2 L_2}{\partial x\partial y}dy\,dx-C_2^2\int_0^b\left[v\frac{\partial^2 L_2}{\partial x}\right]_0^a dy$$

$$+C_2^2\int_0^b\int_0^a v\frac{\partial^2 L_2}{\partial x^2}dx\,dy \qquad (7.1)$$

同様に，(7)式の第2項について計算すると，

$$\int_0^a\int_0^b\frac{1}{\rho}\frac{\partial \sigma_y}{\partial y}L_2\,dy\,dx$$

$$=\frac{1}{\rho}\int_0^a\int_0^b(\sigma_y)' L_2\,dy\,dx$$

$$=\frac{1}{\rho}\int_0^a\left\{\left[\sigma_y L_2\right]_0^b-\int_0^a\sigma_y\frac{\partial L_2}{\partial y}dy\right\}dx$$

ここで，Hookeの法則より

$$\sigma_y=\rho(C_1^2-C_2^2)\frac{\partial u}{\partial x}+\rho\,C_1^2\frac{\partial v}{\partial y}$$

を代入すると，

$$=\frac{1}{\rho}\int_0^a\left\{\left[\sigma_y L_2\right]_0^b-\int_0^b\left\{\rho(C_1^2-2C_2^2)\frac{\partial u}{\partial x}+\rho\,C_1^2\frac{\partial v}{\partial y}\right\}\frac{\partial L_2}{\partial y}dy\right\}dx$$

$$=\frac{1}{\rho}\int_0^a\left[\sigma_y L_2\right]_0^b dx-\int_0^b\int_0^a(C_1^2-2C_2^2)\frac{\partial u}{\partial x}\frac{\partial L_2}{\partial y}dx\,dy-\int_0^a\int_0^b C_1^2\frac{\partial v}{\partial y}\frac{\partial L_2}{\partial y}dy\,dx$$

$$=\frac{1}{\rho}\int_0^a\left[\sigma_y L_2\right]_0^b dx-(C_1^2-2C_2^2)\int_0^b\left\{\left[u\frac{\partial L_2}{\partial y}\right]_0^a-\int_0^a u\frac{\partial^2 L_2}{\partial x\partial y}dx\right\}dy$$

$$-C_1^2\int_0^a\left\{\left[v\frac{\partial L_2}{\partial y}\right]_0^b-\int_0^b v\frac{\partial^2 L_2}{\partial y^2}dy\right\}dx$$

$$=\frac{1}{\rho}\int_0^a[\sigma_y L_2]_0^b dx-(C_1^2-2C_2^2)\int_0^b\left\{\left[u\frac{\partial L_2}{\partial y}\right]_0^a dy+(C_1^2-2C_2^2)\int_0^b\int_0^a u\frac{\partial^2 L_2}{\partial x\partial y}dx\,dy\right.$$

$$-C_1^2\int_0^a\left[v\frac{\partial L_2}{\partial y}\right]_0^b dx+C_1^2\int_0^b\int_0^b v\frac{\partial^2 L_2}{\partial y^2}dy\,dx \qquad (7.2)$$

同様に，(7)式の第 3 項について計算すると，

$$\int_0^b\int_0^a\frac{1}{\rho}\rho\ddot{v}L_2\,dx\,dy$$

$$v=v\sin(\gamma t)$$

$$\frac{\partial v}{\partial t}=\gamma\,v\cos(\gamma t)$$

$$\frac{\partial^2 v}{\partial t^2}=\gamma^2\,v\sin(\gamma t)$$

$$\therefore\int_0^b\int_0^a\{-\gamma^2 v\sin(\gamma t)\}dx\,dy$$

$$=-\gamma^2\int_0^b\int_0^a v\,L_2\,dx\,dy$$

$$=-\gamma^2\int_A^0 v\,L_2\,dA \qquad (7.3)$$

(7.1), (7.2), (7.3)をまとめると，

$$\int_0^a\left[\frac{\sigma_y}{\rho}L_2\right]_0^b dx-C_1^2\int_0^a\left[v\frac{\partial L_2}{\partial y}\right]_0^b dx-(C_1^2-2C_2^2)\int_0^b\left[u\frac{\partial L_2}{\partial y}\right]_0^a dy+\int_A^0 v\left(C_1^2\frac{\partial L_2}{\partial y^2}+C_2^2\frac{\partial L_2}{\partial x^2}-\gamma^2 L_2\right)$$

$$\int_0^b\left[\frac{\tau_{x,y}}{\rho}L_2\right]_0^a dy-C_2^2\int_0^a\left[u\frac{\partial L_2}{\partial x}\right]_0^b dx+(C_1^2-C_2^2)\int_A^0 u\frac{\partial L_2}{\partial x\partial y}dA-C_2^2\int_0^b\left[v\frac{\partial L_2}{\partial x}\right]_0^a dy=0 \qquad (7.4)$$

$L_2=\cos(Mx)\cos(Ny)$ と置いて，

$M=\frac{m\pi}{a},\ N=\frac{n\pi}{b}$　: m,n=1,2,3

とした時，第 1 項から第10項までは次の様に示される。

第 1 項

$$\int_0^a\left[\frac{\sigma y}{\rho}L_2\right]_0^b dx$$

$$=\int_0^a\left[\frac{\sigma_y}{\rho}\cos(Mx)\cos(Ny)\right]_0^b dx$$

$$=\frac{1}{\rho}\int_0^a\cos(Mx)\left\{(\sigma_y)_{y=b}\cos(N\pi)-(\sigma_y)_{y=0}(1)\right\}dx$$

$$=\frac{1}{\rho}\left\{C_m(\sigma_y)_{y=b}(-1)^n-C_m(\sigma_y)_{y=0}\right\} \qquad (7.4.1)$$

第2項

$$-C_1^2\int_0^a\left[v\frac{\partial L_2}{\partial y}\right]_0^b dx$$

$$=-C_1^2\int_0^a[v\{-N\cos(Mx)\sin(Ny)\}]_0^b dx$$

$$=C_1^2\int_0^a[v\sin(Ny)]_0^b N\cos(Mx)dx=0 \qquad (7.4.2)$$

第3項

$$(-C_1^2-2C_2^2)\int_0^b\left[u\frac{\partial L_2}{\partial y}\right]_0^a dy$$

$$=-(C_1^2-2C_2^2)\int_0^b[u\{-N\cos(Mx)\sin(Ny)\}]_0^a dy$$

$$=(C_1^2-2C_2^2)\int_0^b[u\cos(Mx)]_0^a N\sin(Ny)dy$$

$$=(C_1^2-2C_2^2)\int_0^b\{[u_{a,y}](-1)^m-[u_{0,y}]\}N\sin(Ny)dy$$

$$=(C_1^2-2C_2^2)N\{S_n[u_{a,y}](-1)^m-S_n[u_{0,y}]\} \qquad (7.4.3)$$

第4項

$$\int_A^0 v\,C_1^2\frac{\partial^2 L_2}{\partial y^2}dA$$

$$=C_2^2\int_A^0 v\{-N^2\cos(Mx)\cos(Ny)\}dA$$

$$=-C_1^2\,C_mC_n[v]N^2 \qquad (7.4.4)$$

第5項

$$\int_A^0 v\,C_1^2\frac{\partial^2 L_2}{\partial x^2}dA$$

$$=C_2^2\int_A^0 v\{-M^2\cos(Mx)\cos(Ny)\}dA$$

$$=-C_2^2\,C_mC_n[v]M^2 \qquad (7.4.5)$$

第6項

$$\int_A^0 v\gamma^2L_2dA$$

$$=-\gamma^2\int_A^0 v\{\cos(Mx)\cos(Ny)\}dA$$

$$=-\gamma^2\,C_mC_n[v] \qquad (7.4.6)$$

第7項

$$\int_0^b\left[\frac{\tau_{x,y}}{\rho}L_2\right]_0^a d_y$$

$$=\frac{1}{\rho}\int_0^b\left[(\tau_{x,y})\cos(Mx)\cos(Ny)\right]_0^a dy$$

$$=\frac{1}{\rho}\int_0^b\left[(\tau_{x,y})\cos(Mx)\right]_0^a\cos(Ny)\,dy$$

$$=\frac{1}{\rho}\int_0^b\left[(\tau_{x,y})_{x=a}(-1)^m-(\tau_{x,y})_{x=0}\right]\cos(Ny)\,dy$$

$$=\frac{1}{\rho}\left\{\left[C_n(\tau_{x,y})_{x=a}(-1)^m-C_n(\tau_{x,y})_{x=0}\right]\right\} \qquad (7.4.7)$$

第8項

$$-C_2^2\int_0^a\left[u\frac{\partial L_2}{\partial_x}\right]_0^b dx$$

$$=-C_2^2\int_0^a\left[u\{-M\sin(Mx)\cos(Ny)\}\right]_0^b dx$$

$$=C_2^2\int_0^a\left[u\cos(Ny)\right]_0^b M\sin(Mx)\,dx$$

$$=C_2^2\int_0^b\left\{[u_{x,b}](-1)^n-[u_{x,0}]\right\}M\sin(Mx)\,dx$$

$$=C_2^2\left\{S_m[u_{x,b}](-1)^n-S_m[u_{x,0}]\right\}M \qquad (7.4.8)$$

第9項

$$=(C_1^2-C_2^2)\int_A^0 vC_2^2\frac{\partial^r L^2}{\partial x\partial y}\,dA$$

$$=(C_1^2-C_2^2)\int_A^0 u\left\{\sin(Mx)\sin(Ny)\right\}M\,N\,dA$$

$$=(C_1^2-C_2^2)SmSn[u]M\,N \qquad (7.4.9)$$

第10項

$$-C_2^2\int_0^b\left[v\frac{\partial L_2}{\partial x}\right]_0^a dy$$

$$=-C_2^2\int_0^b\left[v\{-M^2\sin(Mx)\cos(Ny)\}\right]_0^a dy$$

$$=C_2^2\int_0^b\left[v\sin(Mx)\right]_0^a M\cos(Ny)\,dy \qquad (7.4.10)$$

(7.4.1)から(7.4.10)をまとめると,

$$\frac{1}{\rho}\left\{C_m(\sigma_y)_{y=b}(-1)^n-C_m(\sigma_y)_{y=0}\right\}+(C_1^2-2C_2^2)N\left\{S_n[u_{a,y}](-1)^m-S_n[u_{0,y}]\right\}$$

$$+\frac{1}{\rho}\left\{C_n(\tau_{x,y})_{x=a}(-1)^m-C_n(\tau_{x,y})_{x=0}\right\}+C_2^2\left\{S_m[u_{x=b}](-1)^n-S_m[u_{x,0}]\right\}M$$

$$-(C_1^2N^2+C_2^2M^2+\gamma^2)C_mC_n[v]+(C_1^2-2C_2^2)S_mS_n[u]MN=0 \qquad (7.5)$$

(7.5)式について

$L_2=\cos(Mx)\cos(Ny)$

$M=\frac{m\pi}{a},\ N=\frac{n\pi}{b}$

(7.5)式において，場合分けして考えると

m=0, n≠0の時,

$$\frac{1}{\rho}\left\{C_0\left(\sigma_y\right)_{y=b}\left(-1\right)^n-C_0\left(\sigma_y\right)_{y=0}\right\}+\left(C_1^2-2C_2^2\right)N\left\{S_n\left[u_{a,y}\right]-S_n\left[u_{0,y}\right]\right\}$$

$$+\frac{1}{\rho}\left[C_n\left(\tau_{x,y}\right)_{x=a}-C_n\left(\tau_{x,y}\right)_{x=0}\right]-\left(C_1^2N^2+\gamma^2\right)C_0C_n\left[v\right]=0$$

m≠0, n=0の時,

$$\frac{1}{\rho}\left\{C_m\left(\sigma_y\right)_{y=b}-C_m\left(\sigma_y\right)_{y=0}\right\}+\frac{1}{\rho}\left[C_0\left(\tau_{x,y}\right)_{x=a}\left(-1\right)^m-C_0\left(\tau_{x,y}\right)_{x=0}\right]$$

$$+C_2^2M\left\{S_m\left[u_{x,b}\right]-S_m\left[u_{x,0}\right]\right\}+\left(C_2^2M^2+\gamma^2\right)C_mC_0\left[v\right]=0$$

m=0, n=0の時,

$$\frac{1}{\rho}\left\{C_0\left(\sigma_y\right)_{y=b}-C_0\left(\sigma_y\right)_{y=0}\right\}+\frac{1}{\rho}\left[C_0\left(\tau_{x,y}\right)_{x=a}-C_0\left(\tau_{x,y}\right)_{x=0}\right]-\gamma^2C_0C_0\left[v\right]=0$$

(6), (7)式を整理すると以下のように示される。

$$-C_1^2\left\{S_n\left[u_{a,y}\right]\left(-1\right)^m-S_n\left[u_{0,y}\right]\right\}M-C_2^2\left\{S_m\left[u_{x,b}\right]\left(-1\right)^n-S_m\left[u_{x,0}\right]\right\}N$$

$$-S_mS_n\left[u\right](C_1^2M^2+C_2^2N^2+\gamma^2)+C_mC_n\left[v\right](C_1^2-C_2^2)MN=\gamma^2S_mS_n\left[u_0\right] \qquad (8)$$

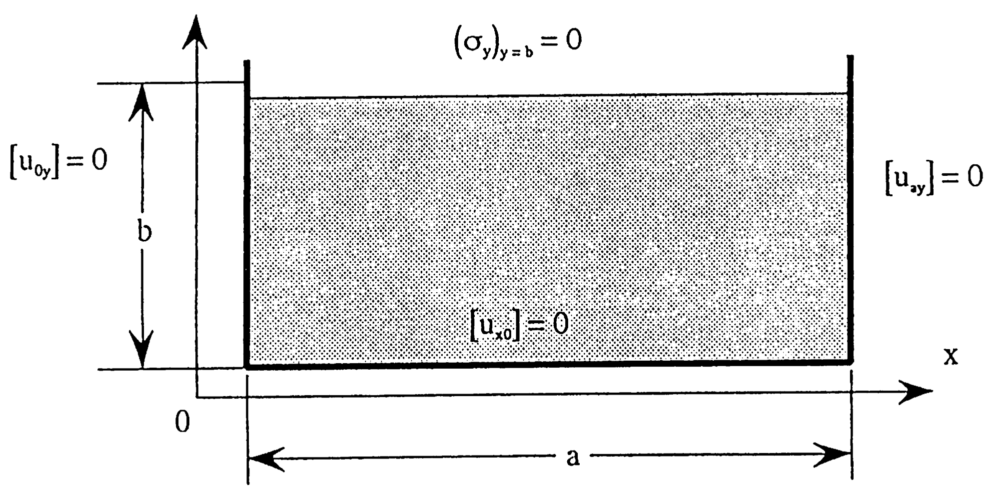
$$\frac{1}{\rho}\left\{C_m\left[\left(\sigma_y\right)_{y=b}\right]\left(-1\right)^n-C_m\left[\left(\sigma_y\right)_{y=0}\right]\right\}+\left(C_1^2-2C_2^2\right)N\left\{S_n\left[u_{a,y}\right]\left(-1\right)^m-S_n\left[u_{0,y}\right]\right\}$$

$$+\frac{1}{\rho}\left\{C_n\left[\left(\tau_{x,y}\right)_{x=a}\right]\left(-1\right)^m-C_n\left[\left(\tau_{x,y}\right)_{x=0}\right]\right\}+C_2^2\left\{S_m\left[u_{x,b}\right]\left(-1\right)^n-S_m\left[u_{x,0}\right]\right\}M$$

$$-\left(C_1^2N^2+C_2^2M^2+\gamma^2\right)C_mC_n\left[v\right]+\left(C_1^2-C_2^2\right)S_mS_n\left[u\right]MN=0 \qquad (9)$$

ここで,
$S_mS_n[u]=U_{m,n}$, $C_mC_n[v]=V_{m,n}$
とおく

更に，以下の図－2の境界条件を考慮する。
- $[u_{0,y}]$, $[u_{a,y}]$については，壁面なので変位はない。
- $[u_{x,0}]$については，底板と砂が一体となって働くと仮定している。
- $(\sigma_y)_{y=b}$については，上面では応力は働かない。



図－2 境界条件

$$\begin{vmatrix} C_1^2M^2+C_2^2N^2+\gamma^2 & -(C_1^2-C_2^2)MN \\ -(C_1^2-C_2^2)MN & C_1^2N^2+C_2^2M^2+\gamma^2 \end{vmatrix}\begin{Bmatrix} U_{m,n} \\ V_{m,n} \end{Bmatrix}=$$

$$\begin{vmatrix} -\gamma^2 S_mS_n[u_0]-N(-1)^nC_2^2S_m[u_{x,b}] \\ -\frac{1}{\rho}\left[C_m(\sigma_y)_{y=0}\right]+M(-1)^nC_2^2S_m[u_{x,b}]+\frac{1}{\rho}\left\{\left[C_n(\tau_{xy})_{x=a}(-1)^m-C_n(\tau_{x,y})_{x=0}\right]\right\} \end{vmatrix}$$

ここで,

$$\frac{C_2^2}{C_1^2}=k,\ \frac{\gamma^2}{C_1^2}=p^2,\ F_{m,n}=p^2S_mS_n[U_0],\ S_m[U_{x,b}]=A_m,\ \frac{1}{\rho}C_m\left[(\sigma y)_{g=0}\right]=B_m$$

とすると

$x=0$, aで, $t_{x,y}=0$ and $y=b$で, $\sigma y=0$, $t_{x,y}=0$, $y=0$で, $v=0$, $u=0$

であり，(10)式と整理される。

$$\begin{vmatrix} M^2+kN^2+p^2 & -(1-k)MN \\ -(1-k)MN & kM^2+N^2+p^2 \end{vmatrix}\begin{Bmatrix} U_{m,n} \\ V_{m,n} \end{Bmatrix}=\begin{vmatrix} -F_{m,n}-kA_m(-1)^nN \\ -B_m+kA^m(-1)^nM \end{vmatrix} \qquad (10)$$

$$(M^2+kN^2+p^2)=A_1,\quad \left\{-(1-k)MN\right\}=A_2,B_1,\quad (kM^2+N^2+p^2)=B_2$$

$$\left\{-F_{m,n}-kA_m(-1)^nN\right\}=C_1,\quad \left\{-B_m+kA_m(-1)^nM\right\}=C_2$$

$$U_{m,n}=\frac{C_1B_2-C_2A_1}{A_1B_2-A_2B_1}$$

$$=\frac{\left\{-F_{m,n}-kA_m(-1)^nN\right\}(kM^2+N^2+p^2)-\left\{-B_m+kA_m(-1)^nM\right\}\left\{-(1-k)MN\right\}}{(M^2+kN^2+p^2)(kM^2+N^2+p^2)-\left\{-(1-k)MN\right\}\left\{-(1-k)MN\right\}}$$

分子

$$\left\{-F_{m,n}(kM^2+N^2+p^2)-kA_m(-1)^nN(kM^2+N^2+p^2)\right\}$$

$$-\left[-B_m\left\{-(1-k)MN\right\}+kA_m(-1)^nM\left\{-(1-k)MN\right\}\right]$$

$$=-F_{m,n}(kM^2+N^2+p^2)-kA_m(-1)^nN(kM^2+N^2+p^2)-B_m(1-k)MN+kA_m(-1)^nM(1-k)MN$$

$$=-F_{m,n}(kM^2+N^2+p^2)-B_m(1-k)MN+kA_m(-1)^nN\left\{-(kM^2+N^2+p^2)+M^2(1-k)\right\}$$

$$=-F_{m,n}(kM^2+N^2+p^2)-B_m(1-k)MN+kA_m(-1)^nN\left\{-kM^2+N^2+p^2+M^2-kM^2\right\}$$

$$=-F_{m,n}(kM^2+N^2+p^2)-B_m(1-k)MN+kA_m(-1)^nN\left\{(1-2k)M^2-N^2-p^2\right\}$$

分母

$$(M^2+kN^2+p^2)(kM^2+N^2+p^2)$$

$$=kM^4+kM^2N^2+kM^2p^2+M^2N^2+kN^4+N^2p^2+M^2p^2+kN^2p^2+p^4$$

$$=k(M^4+N^4)+M^2N^2(k^2+1)+p^2(1+k)(M^2+N^2)+p^4$$

$$\left\{-(1-k)MN\right\}^2$$
$$=M^2N^2-2kM^2N^2+k^2M^2N^2$$
$$=M^2N^2(1+k^2)-2kM^2N^2$$
$$\therefore (M^2+kN^2+p^2)(kM^2+N^2+p^2)-\left\{-(1-k)MN\right\}\left\{-(1-k)MN\right\}$$
$$=k(M^4+N^4)+M^2N^2(k^2+1)+p^2(1+k)(M^2+N^2)+p^4-M^2N^2(1+k^2)-2kM^2N^2$$
$$=k(M^4+N^4)+p^2(M^2+N^2)+kp^2(M^2+N^2)+p^4+2kM^2N^2$$
$$=k(M^2+N^2)^2+p^2(M^2+N^2)+kp^2(M^2+N^2)+p^4$$

ここで,

$$A=M^2+N^2$$

とおく

$$=kA^2+p^2kA+p^2A+p^4$$
$$=(A+p^2)(kA+p^2)$$
$$\therefore (M^2+N^2+p^2)\left\{k(M^2+N^2)+p^2\right\}$$
$$=(M^2+N^2+p^2)(kM^2+kN^2+p^2)$$

ここで,

$$\Omega_{m,n}=k(M^2+N^2+p^2)(kM^2+kN^2+p^2)=kHH'$$

where $H=M^2+N^2+p^2$, $H'=kM^2+kN^2+p^2$

$$\therefore U_{m,n}=\frac{-F_{m,n}(kM^2+N^2+p^2)-B_m(1-k)MN+kA_m(-1)^n N\left\{(1-2k)M^2-N^2-p^2\right\}}{\Omega_{m,n}}$$

同様に,

$$V_{m,n}=\frac{c_2a_1-c_1b_1}{a_1b_2-a_2b_1}$$
$$=\frac{\left\{-B_m+kA_m(-1)^n M\right\}(M^2+kN^2+p^2)(kM^2+N^2+p^2)-\left\{-F_{m,n}-kA_m(-1)^n N-\left\{-(1-k)MN\right\}\right.}{(M^2+kN^2+p^2)(kM^2+N^2+p^2)-\left\{-(1-k)MN\right\}\left\{-(1-k)MN\right\}}$$

分子

$$=-B_m(M^2+kN^2+p^2)+kA_m(-1)^n M(M^2+kN^2+p^2)-\left[-F_{m,n}\left\{-(1-k)MN\right\}\right]$$
$$-kA_m(-1)^n N\left\{-(1-k)MN\right\}$$
$$=-B_m(M^2+kN^2+p^2)+kA_m(-1)^m M(M^2+kN^2+p^2)-F_{m,n}(1-k)MN$$
$$-kA_m(-1)^n N(1-k)MN$$
$$=-F_{m,n}(1-k)MN-B_m(M^2+kN^2+p^2)+kA_m(-1)^n M\left\{(M^2+kN^2+p^2)-N^2(1-k)\right\}$$

$$=-F_{m,n}(1-k)MN-B_m(M^2+kN^2+p^2)+kA_m(-1)^n M(M^2+kN^2+p^2-N^2+kN^2)$$

$$=-F_{m,n}(1-k)MN-B_m(M^2+kN^2+p^2)+kA_m(-1)^n M(M^2-N^2+2kN^2+p^2)$$

$$=-F_{m,n}(1-k)MN-B_m(M^2+kN^2+p^2)+kA_m(-1)^n M\{M^2-(1-2k)N^2+p^2\}$$

$$=k(M^4+N^4)+p^2(M^2+N^2)+kp^2(M^2+N^2)+p^4+2kM^2N^2$$

分母

$$(M^2+kN^2+p^2)(kM^2+N^2+p^2)$$

$$=kM^4+kM^2N^2+kM^2p^2+M^2N^2+kN^4+N^2p^2+M^2p^2+kN^2p^2+p^4$$

$$=k(M^4+N^4)+M^2N^2(k^2+1)+p^2(1+k)(M^2+N^2)+p^4$$

$$\{-(1-k)MN\}^2$$

$$=M^2N^2-2kM^2N^2+k^2M^2N^2$$

$$=M^2N^2(1+k^2)-2kM^2N^2$$

$$\therefore (M^2+kN^2+p^2)(kM^2+N^2+p^2)-\{-(1-k)MN\}\{-(1-k)MN\}$$

$$=k(M^4+N^4)+M^2N^2(k^2+1)+p^2(1+k)(M^2+N^2)+p^4-M^2N^2(1+k^2)-2kM^2N^2$$

$$=k(M^4+N^4)+p^2(M^2+N^2)+kp^2(M^2+N^2)+p^4+2kM^2N^2$$

$$=k(M^2+N^2)^2+p^2(M^2+N^2)+kp^2(M^2+N^2)+p^4$$

ここで,

$$A=M^2+N^2$$

とおく

$$=kA^2+p^2kA+p^2A+p^4$$

$$=(A+p^2)(kA+p^2)$$

$$\therefore (M^2+N^2+p^2)\{k(M^2+N^2)+p^2\}$$

$$=(M^2+N^2+p^2)(kM^2+kN^2+p^2)$$

ここで,

$$\Omega_{m,n}=k(M^2+N^2+p^2)(kM^2+kN^2+p^2)=kHH'$$

where $H=M^2+N^2+p^2$, $H'=kM^2+kN^2+p^2$

$$\therefore U_{m,n}=\frac{-F_{m,n}(1-k)MN-B_m(kM^2+N^2+p^2)+kA_m(-1)^n M\{M^2-(1-2k)M^2-(1-2k)N^2+p^2\}}{\Omega_{m,n}}$$

よって，逆変換を施すことによって平面歪み問題としての解が，以下のように求められる。

(2.1) 水平変位

$$u=\sum_{m}\frac{\{1-(-1)^{m}\}}{2}\sin(Mx)$$

$$\Big[\frac{4u_0}{b}\frac{be}{m\pi}\big\{(1-Q_m(\eta))\big\}\sin\omega t$$

$$-\frac{2hA_me\pi}{b^2p^2}\frac{b}{\pi}\big\{2(me)^2(R'_m(\eta))-p'^2R'_m(\eta)\big\}$$

$$+\frac{2\beta_m e}{bp^2}\frac{b}{\pi}(me)\big\{R_m(1-\eta)-R'_m(1-\eta)\big\}\Big]$$

$$+u_0\sin(\omega t)$$

(2.2) 鉛直変位

$$v=\sum_{m}\frac{\{1-(-1)\}}{2}\cos(Mx)$$

$$\Big[\frac{4u_0}{b}\frac{be}{\pi}\Big(\frac{\varphi'_m(\eta)}{\beta'_m}-\frac{\varphi_m(\eta)}{\beta_m}\Big)+\frac{2hA_me\pi}{b^2p^2}\frac{b}{\pi}(me)\big\{2(\beta'_m\varphi'_m(\eta)-\beta_m\varphi_m(\eta))+\frac{p'^2}{\beta'_m}\varphi'_m(\eta)\big\}$$

$$-\frac{2\beta_m e}{bp^2}\frac{b}{\pi}\Big\{\frac{(me)}{\beta'_m}\varphi'_m(1-\eta)-\beta_m\varphi_m(1-\eta)\Big\}\Big]$$

uは垂直壁で，$u_0\sin\omega t$の変位，底面で$v=0$, $u={}_0\sin\omega t$。

$$p^2=\frac{\omega^2b^2}{C_1^2\pi^2},\quad p'^2=\frac{\omega^2b^2}{C_2^2\pi^2},\quad h=\frac{C_1^2}{C_2^2},\quad C_1^2=\frac{E}{\rho},\quad C_2^2=\frac{G}{\rho}$$

変位の一次微分とRotationは以下の様に示される。

$$\frac{\partial u}{\partial y}=\sum\frac{\{1-(-1)^m\}}{2}\sin(Mx)\Big[\frac{4u_0}{b}\frac{1}{m}\{\varphi_m(\eta)\frac{(me)^2}{\beta_m}-\varphi'_m(\eta)\beta'_m\}$$

$$-\frac{2hA_me\pi}{b^2p^2}\{2(me)^2(\varphi'_m(\eta)\beta'_m-\varphi_m(\eta)\beta_m)-p'^2\beta'_m\varphi_m(\eta)\}$$

$$+\frac{2\beta_m e}{bp^2}(me)\{\varphi'_m(1-\eta)\beta'_m-\varphi_m(1-\eta)\beta_m\}\Big]$$

$$\frac{\partial v}{\partial x}=-\sum\frac{\{1-(-1)^m\}}{2}\sin(Mx)\frac{4u_0}{b}(me)^2\Big\{\frac{\varphi'_m(\eta)}{\beta'_m}-\frac{\varphi_m(\eta)}{\beta_m}\Big\}$$

$$+\frac{2hA_me\pi}{b^2p^2}(me)^2\Big\{2(\varphi'_m(\eta)\beta'_m-\varphi_m(\eta)\beta_m)+\frac{p'^2}{\beta'_m}\varphi'_m\Big\}$$

$$-\frac{2\beta_m e}{bp^2}(me)\Big\{\frac{(me)^2}{\beta'_m}\varphi'_m(1-\eta)-\varphi_m(1-\eta)\beta_m\Big\}\Big]$$

$$\therefore\omega=\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}$$

$$=\sum\frac{\{1-(-1)^m\}}{2}\sin(Mx)\Big[\frac{4u_0}{b}\varphi'_m(\eta)\frac{p'^2}{m\beta'_m}$$

$$+\frac{2hA_me\pi}{b^2p^2}\Big\{\frac{2(me)^2-p'^2}{\beta'_m}\varphi'_m(\eta)\Big\}-\frac{2\beta_m e}{bp^2}\frac{me}{\beta'_m}p'^2\varphi'_m(1-\eta)\Big]$$

（２．３）x方向の応力

$$\frac{\sigma_x}{\rho C_1^2}=\sum_m \frac{\{1-(-1)^m\}}{2}\cos(Mx)$$

$$\Big[\frac{4u_0}{b}e\Big\{(1-Q_m(\eta))\frac{p^2}{\beta^2{}_m}+2h(Q'_m(\eta)-Q_m(\eta)\Big\}$$

$$+\frac{2hA_m e\pi}{b^2p^2}me\Big\{-4(me)^2(R'_m(\eta)-R_m(\eta))+2p^2R'_m(\eta)+2(1-2h)p^2R_m(\eta)\Big\}$$

$$+\frac{2\beta_m e}{bp^2}\Big\{2h(me)^2\{R_m(1-\eta)-R'_m(1-\eta)\}+p^2(1-2h)R_m(1-\eta)\Big\}\Big]$$

（２．４）せん断応力

$$\frac{\tau_{x,y}}{\rho C_2^2}=\sum \frac{\{1-(-1)\}}{2}\sin(Mx)$$

$$\Big[\frac{4u_0}{b}\Big\{2me^2\Big(\frac{\varphi'_m(\eta)}{\beta'_m}-\frac{\varphi_m(\eta)}{\beta_m}\Big)+\frac{p'^2}{m\beta'_m}\Big\}-\frac{2hA_m e\pi}{b^2p^2}\Big\{4(me)^2(\beta'_m\varphi'_m(\eta)-\beta_m\varphi_m(\eta))+\frac{p'^4}{\beta'_m}\varphi'_m(\eta)\Big\}$$

$$+\frac{2\beta_m e}{bp^2}me\Big\{\beta'_m\varphi'_m(1-\eta)-2\beta_m\varphi_m(1-\eta)+\frac{(me)^2}{\beta'_m}\varphi'_m(1-\eta)\Big\}\Big]$$

ここで，

$$Q_m(\eta)=\frac{\cosh\Big\{\beta_m\pi\Big(\frac{1}{2}-\eta\Big)\Big\}}{\cosh\Big(\frac{\beta_m\pi}{2}\Big)} \qquad \left(=\frac{\cos\Big\{\beta_m\pi\Big(\frac{1}{2}-\eta\Big)\Big\}}{\cos\Big(\frac{\beta_m\pi}{2}\Big)}\right)$$

$$R_m(\eta)=\frac{\sinh(\beta_m\pi\eta)}{\sinh(\beta_m\pi)} \qquad \left(=\frac{\sin(\beta_m\pi\eta)}{\sin(\beta_m\pi)}\right)$$

$$R_m(1-\eta)=\frac{\sinh\{\beta_m\pi(1-\eta)\}}{\sinh(\beta_m\pi)} \qquad \left(=\frac{\sin\{\beta_m\pi(1-\eta)\}}{\sin(\beta_m\pi)}\right)$$

$$\varphi_m(\eta)=\frac{\cosh(\beta_m\pi\eta)}{\sinh(\beta_m\pi)} \qquad \left(=\frac{\cosh(\beta_m\pi\eta)}{\sinh(\beta_m\pi)}\right)$$

$$\varphi_m(1-\eta)=\frac{\cos\{\beta_m\pi(1-\eta)\}}{\sin(\beta_m\pi)} \qquad \left(=\frac{\cos\{\beta_m\pi(1-\eta)\}}{\sin(\beta_m\pi)}\right)$$

$$\phi_m(\eta)=\frac{\sinh\Big\{\beta_m\pi\Big(\frac{1}{2}-\eta\Big)\Big\}}{\cosh\Big(\frac{\beta_m\pi}{2}\Big)} \qquad \left(=\frac{\sin\Big\{\beta_m\pi\Big(\frac{1}{2}-\eta\Big)\Big\}}{\cos\Big(\frac{\beta_m\pi}{2}\Big)}\right)$$

$$Q'_m(\eta)=\frac{\cosh\Big\{\beta'_m\pi\Big(\frac{1}{2}-\eta\Big)\Big\}}{\cosh\Big(\frac{\beta'_m\pi}{2}\Big)} \qquad \left(=\frac{\cos\Big\{\beta'_m\pi\Big(\frac{1}{2}-\eta\Big)\Big\}}{\cos\Big(\frac{\beta'_m\pi}{2}\Big)}\right)$$

$$R'_m(\eta)=\frac{\sinh(\beta'_m\pi\eta)}{\sinh(\beta'_m\pi)} \qquad \left(=\frac{\sin(\beta'_m\pi\eta)}{\sin(\beta'_m\pi)}\right)$$

$$R'_m(1-\eta)=\frac{\sinh\{\beta'_m\pi(1-\eta)\}}{\sinh(\beta'_m\pi)} \qquad \left(=\frac{\sin\{\beta'_m\pi(1-\eta)\}}{\sin(\beta'_m\pi)}\right)$$

$$\varphi'_m(\eta)=\frac{\cosh(\beta'_m\pi\eta)}{\sinh(\beta'_m\pi)} \qquad \left(=\frac{\cosh(\beta'_m\pi\eta)}{\sinh(\beta'_m\pi)}\right)$$

$$\varphi'_m(1-\eta)=\frac{\cos\{\beta'_m\pi(1-\eta)\}}{\sin(\beta'_m\pi)} \qquad \left(=\frac{\cos\{\beta'_m\pi(1-\eta)\}}{\sin(\beta'_m\pi)}\right)$$

$$\phi'_m(\eta)=\frac{\sinh\left\{\beta'_m\pi\left(\frac{1}{2}-\eta\right)\right\}}{\cosh\left(\frac{\beta'_m\pi}{2}\right)} \qquad \left(=\frac{\sin\left\{\beta'_m\pi\left(\frac{1}{2}-\eta\right)\right\}}{\cos\left(\frac{\beta'_m\pi}{2}\right)}\right)$$

※なお，（　）内は，imaginary。
ここで，

$$\beta^2=(me)^2-p^2$$

$$\beta'^2=(me)^2-p'^2 \qquad \left(p'^2=\frac{p^2}{h}\right)$$

$$M=\frac{m\pi}{a}, \qquad \eta=\frac{y}{b}, \qquad e=\frac{b}{a}, \qquad h=\frac{C_2^2}{C_1^2}=\frac{G}{2G+\lambda}=\frac{1-2\upsilon}{2(1-\upsilon)}$$

G: せん断弾性係数，υ: ポアソン比，λ: ラーメン定数

境界未知数
(1) 境界条件

$$\begin{cases}(\tau_{x,y})_{\eta=1}=0\\ (v)_{\eta=0}=0\end{cases}$$

(2) 境界未知数(X_m，Y_m)

$$\begin{cases}-F_1X_m+F_2Y_m=-E_1\\ F_2X_m-F_3Y_m=E_2\end{cases}$$

$$\frac{2hA_me\pi}{p^2b^2}=\frac{4u_0}{b}X_m,\ \frac{2B_me}{p^2b^2}=\frac{4u_0}{b}Y_m$$

ここで

$$F_1=4(me)^2\{B_m'\phi_m'(1)-B_m\phi_m(1)\}+p'^4\frac{\phi_m'(1)}{B_m'}$$

$$F_2=me\left[2\{B_m'\phi_m'(0)-B_m\phi_m(0)\}\right]+p'^2\frac{\phi_m'(0)}{B_m'}$$

$$F_3=(me)^2\frac{\phi_m'(1)}{B_m'}-B_m\phi_m(1)$$

$$E_1=2me^2\left\{\frac{\phi_m'(0)}{B_m'}-\frac{\phi_m(0)}{B_m'}\right\}-\frac{p^2}{m}\frac{\phi_m'(0)}{B_m'}$$

$$E_2=e\left(\frac{\phi_m(0)}{B_m}-\frac{\phi_m'(0)}{B_m'}\right)$$

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