Resonant Transmission of Two Acoustic Phonon Modes in a Superlattice with (111) Interfaces

Hatsuyoshi KATO

(Received November 18 1996)

Tomakomai National College of Technology
443, Nishikioka, Tomakomai, Hokkaido 059-12, Japan

Abstract

Oscillations of transmission rates of acoustic phonons are studied in a superlattice with (111) interfaces. The oscillations are enhanced at frequencies that satisfy the resonant condition of transmission. In this condition, reflections from the interfaces are neglectable. Structure factors are introduced for transmission and reflection of acoustic phonons to explain this phenomenon. Transfer matrix with reduced dimension is also introduced.The oscillation is analogous to the Pendellösung effect of electron beams and the Borrmann effect of X-rays.

1. Introduction

Recently there are many works about acoustic phonons in various superlattices. Tamura, Hurley and Wolfe showed the phonon images of transmitted phonons through a superlattice (SL) both theoretically and experimentally.[1] Tamura found the resonant transmission of acoustic phonons in a SL,[2] that is analogous to the resonant tunneling of electrons. In the previous paper, we have studied oscillations of transmission rates for acoustic phonons against a number of layers.[3]

One of the oscillations are analogous to the Pendellösung effect of electron beams in electron microscopy[4], and the Borrmann effect of X-rays[5]. In these cases, energy of reflected and transmitted beams oscillates when the beams propagate through very perfect crystals. The oscillations of transmission rates for phonons mean that there exist strong energy exchanges between two modes of acoustic phonons, when the phonons are incident obliquely to the interfaces of the SL. In the present paper, we shall try to show the origin of these oscillations in a superlattice with anisotropic layers.

Figure 1 shows the structure of the SL. Unit cell of the SL consists of 150-monolayer AlAs and 60-monolayer GaAs, and their interfaces are assumed as the (111) planes as in Fig.1(a). The monolayer has a thickness of 0.326nm. Hence, the thickness of the unit cell becomes D=68.5nm. Figure 1(b) shows the structure of the SL with five unit cells. A substrate is a layer of GaAs and a detector layer is of AlAs. Even if we change the number of the unit cells, the substrate and the detector layers are still supposed to be the same materials. In the figure, the x, y, and z axes are placed to the directions $[11\bar{2}]$, $[\bar{1}10]$, and $[111]$, respectively.

Generally there are three modes of phonons in crystals.[6] Howerver, we consider only two modes which couple each other on $(\bar{1}10)$ saggital plane: modes S and L (Quasi-Shear and Quasi-Longitudinal). The other mode SH (Pure-Shear) does not couple with these two modes even if we consider oblique incidence of phonons to the interfaces. Figure2(a) shows slowness curves in the saggital plane. Figure2(b) depicts polarization angles of the phonon modes. Each curve has two-fold symmetry about the y axis.

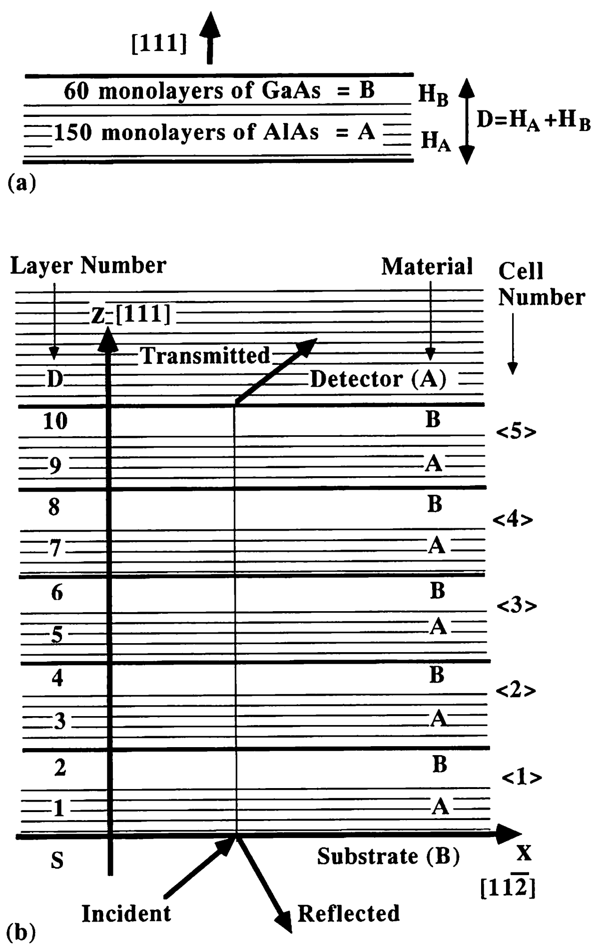

Fig.1. (a) A unit cell consists of 60-monolayer GaAs and 150-monolayer AlAs, and (b) structure of superlattice.

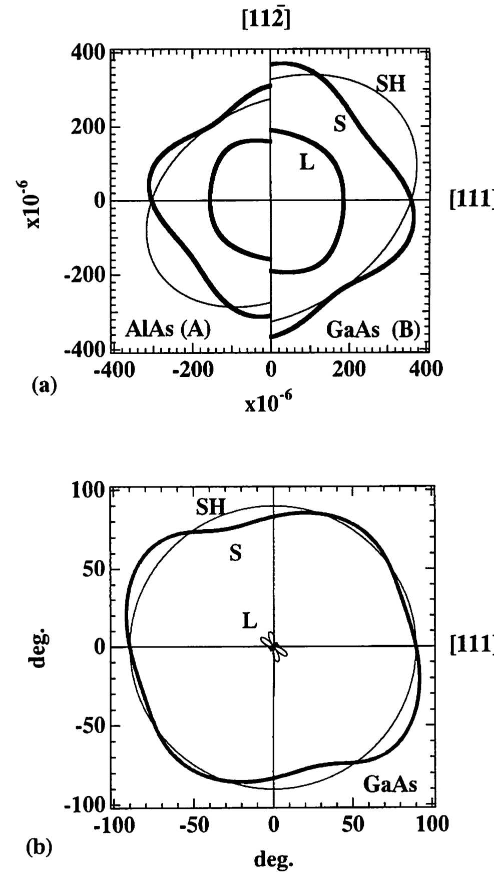

Fig.2. (a) Slowness curves. In right hand, there are slowness curves of GaAs. In left hand, those of AlAs. Unit of the axes is s/m. Polarization angles of the phonons are depicted. Each radius represents the porization angle from the direction of the phonon wave vector.

2. Structure Factors

To explain frequency dependence of transmission and reflection rates of phonons, we introduce two kinds of structure factors. One is for reflections and the other is for trnsmissions.

For the reflections, there are two structure factors: with/without mode conversions. They are expressed as follows:

$$S_{JK}^{<R>} = r_{BA} + (r_{BA} + r_{AB} e^{-i\theta_{JK}^{(B)}}) \frac{1-e^{iN(\theta_{JK}^{(A)}+\theta_{JK}^{(B)})}}{1-e^{i(\theta_{JK}^{(A)}+\theta_{JK}^{(B)})}}, \quad (1a)$$

$$\theta_{JK}^{(j)} \equiv \zeta_{J+}^{(j)} H_j + \zeta_{K-}^{(j)} H_j, \quad (1b)$$

where N is a number of unit cells, J, K = S or L; j, k = A or B. Magnitudes of z-components of a phonon wave vector with mode J in the layer j are $\zeta_{J\pm}^{(j)} > 0$ (the sign '+' represents phonons to propagate to the positive z-direction, and the sign '−' is of reflected phonons propagating to the negative z-direction). H_j is a thickness of the layer j. The quantity r_{jk} is an amplitude reflection coefficient at the interface of the layers j and k. (It depends also on the modes J and K.) If $J=K$, Eq.(1) means the structure factor without mode conversions. If $J \neq K$, it is the structure factor with mode conversions. From Eq.(1), we get the Bragg condition for reflections in a SL as follows:

$$\zeta_{J+}^{(A)} H_A + \zeta_{J+}^{(B)} H_B + \zeta_{K-}^{(B)} H_B + \zeta_{K-}^{(A)} H_A = 2\pi m, \quad (2)$$

where m is an integer. If this condition is satisfied, $|S_{JK}^{<R>}|^2$ has a peak, and reflections of the phonons are enhanced.

For phonon transmissions with mode conversions, we get the following structure factor:

$$S_{JK}^{<T>} = t_{BA} + (t_{BA} + t_{AB} e^{i(\theta_K^{(B)} - \theta_J^{(B)})}) \times e^{i(\theta_J - \theta_K)} \frac{1 - e^{iN(\theta_J - \theta_K)}}{1 - e^{i(\theta_J - \theta_K)}}, \quad (3a)$$

$$\theta_J^{(j)} \equiv \zeta_{J+}^{(j)} H_j, \quad (3b)$$

$$\theta_J \equiv \theta_J^{(A)} + \theta_J^{(B)}, \quad (3c)$$

where $J \neq K$, because Eq.(3) is the kinematic structure factor derived from perturbation and there are no significant meanings if $J=K$. The quantity t_{jk} is an amplitude transmission coefficient at the interface of the layers j and k. (Be careful again that it depends also on the modes J and K like the reflection coefficients.) We should note that $|S_{JK}^{<T>}|^2$ has a peak at the same condition for both of the mode conversions (from S to L and from L to S). From Eq.(3), we get the condition for enhancement of phonon transmissions with mode conversions as follows:

$$\zeta_{S+}^{(A)} H_A + \zeta_{S+}^{(B)} H_B = \zeta_{L+}^{(A)} H_A + \zeta_{L+}^{(B)} H_B + 2\pi m \equiv \theta, \quad (4)$$

where m is an integer. We call this condition as a resonant condition of transmission in later sections. Equation (4) tells that the resonant frequency decreases if the thickness of the unit cell is increased, because the wave number is directly proportional to the phonon frequencies.

Frequency dependence of the structure factors are in Fig.3. Figure 3(a) is for S-mode incident to the interfaces with an angle 20 degrees between the incident wave vector and the normal of the interface. The angle is also the same in every layer B in Fig.1(b). Plots of positive values are of the transmission with mode conversion from S to L. The negative is of the reflection with/without mode conversions: both from S to L and from S to S. Figure 3(b) is for L-mode incidence. The angle between propagation direction of L-mode phonons and the normal of the interface is 34.2 degrees in the substrate and the layer B. Note that at peak frequency of the transmission rate all structure factors for reflection vanish.

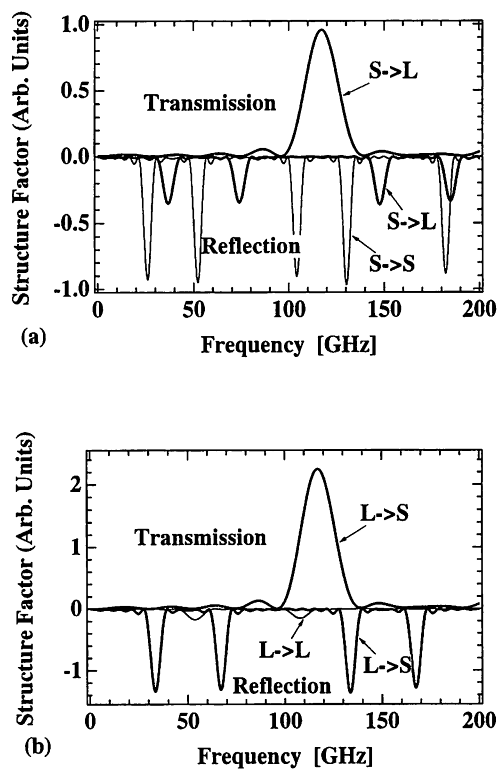

Fig.3. Structure factors in arbitary units. (a) For S mode incidence. (b) For L mode incidence.

3. Frequency and Angular Dependence

The transfer matrix method is used to get transmission and reflection rates. Because we are considering only two modes, we need 4×4 matrices. (Generally, 6×6 matrices must be considered.[1])

Figure 4(a) and (b) show transmission and reflection rates given by the transfer matrix method. T_{JK} is a transmission rate of phonon energy in the detector layer after the mode conversion from J to K. R_{JK} is a reflection rate of phonon energy in the substrate. The transmission rates are expressed as positive values, and the reflection rates are as negative values in the figure.

Even if the Bragg condition is satisfied, there are some cases that the structure factors prohibit the reflections because of the extinction rule. Transmission rate without mode conversions is not in the figure. However, it can be gotten by the energy conservation.

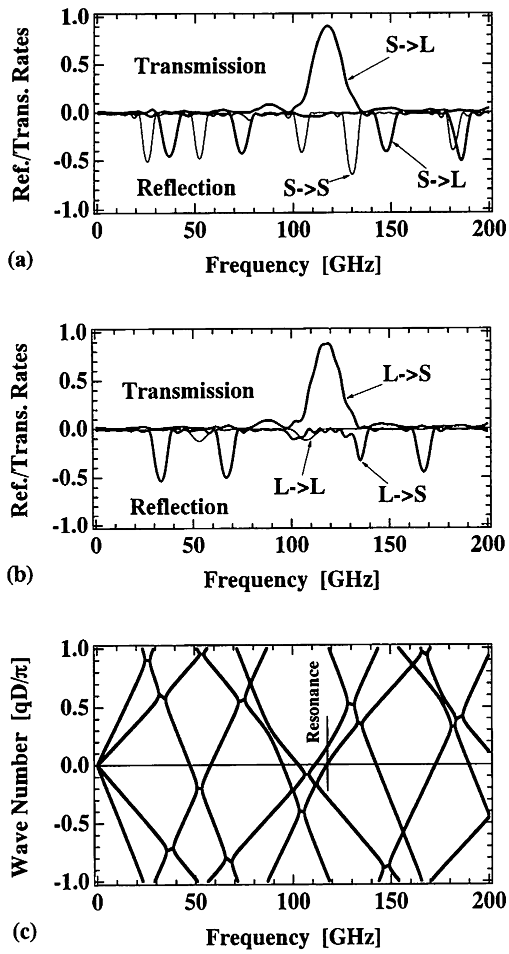

Fig.4. Reflection/Transmission rates, and dispersion relation. (a) Reflection rates are plotted as negative values, transmission rate is as positive. The incident mode is S and the incident angle is 20 degrees. (b) For L-mode incidence with the angle 34.2 deg. (c) Dispersion relations. The incident angles are 20 degrees for S-mode and 34.2 degrees for L-mode.

Figure 4(c) shows the dispersion relation for Fig.4(a) and (b). (Note that the branches are not symmetrical about zone center. This stems from the anisotropy of each layer.) The wave number is normalized by the factor D/π . Vertical line represents a frequency (117.6 GHz) that satisfies the resonant condition of Eq.(4). Around the frequency, there are anti-crossing braches in the dispersion relation. At the stop bands in figure (c), the reflection rates in figure(a) and (b) have main peaks. Comparing Fig.3 with Fig.4, we can notice that there is precise correspondence between the peaks of the structure factors and the transmission/reflection rates.

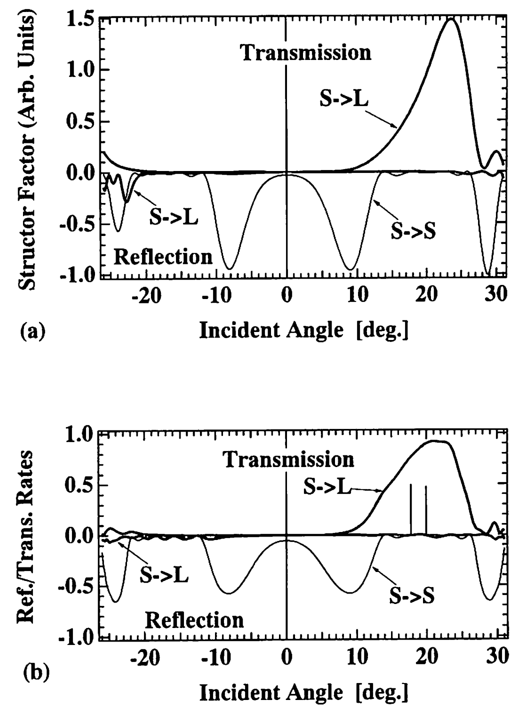

Fig.5. Angular dependence. (a) Structure factors for S-mode incidence. Frequency is 117.6 GHz. Structure factors for L-mode incidence is not shown in this figure. However, the factor for transmission with mode conversion has the same shape and its peak is at the same angle as of the S-mode incidence. (b) Reflection and transmission rates. The vertical lines are at the resonant frequencies.

Figure 5 shows an angular dependence. The structure factors are plotted in Fig.5(a), and the transmission/reflection rates are in Fig.5(b). Vertical lines in Fig.5(b) are at frequencies which satisfy the resonant condition. However, the stractor for transmission and the transmission rate do not have peaks at these frequencies. This is explained by the prefactor of the structure factor for transmission. The prefactor in Eq.(3) partly suppresses the transmission.
Therefore, the peaks appear at a frequency different from the resonant frequencies given by Eq.(4). Instead of this feature, the structure factor, Eq.(3), gives the precise angular dependence of transmission and reflection rates as in Fig.5(b).

4. Oscillations of Transmission Rates

Energy of phonons are exchanged while the phonons of the two modes get through the interfaces of the SL. This effect is enhanced at the frequencies of the resonant condition. Figure 6 shows the transmission rates against number of cells at this condition. They oscillate with a period $\Delta N=13.6$, when a frequency is 117.6 GHz and an angle of S-mode incidence is 20 degrees. The oscillations can be explained as follows:

"At the frequency of the resonant condition, the structure factors for all reflections become zero as in Fig.3. This makes the incident phonons get through the interfaces of the SL without any reflections. Simultaneously the structure factor for transmissions with the mode conversion has peaks as in Fig.3. This makes the phonons exchange the energy between the two modes. Hence, the transmitted phonons must get through the SL without any reflections and with mode conversions (between the modes S and L). Therefore, there appear the oscillations of transmission rates against the number of cells."

According to the numerical calculations, the energy reflecting from the SL is less than 5% of the total energy at the resonant condition.Therefore, we can neglect reflections of phonons from the interfaces. Using this feature, we can explain the oscillations of transmission with 2 × 2 matrices. The relation between amplitudes of displacements in the detector and the substrate is expressed by the transfer matrix method as follows:

$$\begin{bmatrix} A_S^{(D)} \\ A_L^{(D)} \end{bmatrix} = \left[M^{(A)}\right]^{-1} T^N M^{(B)} \begin{bmatrix} A_S^{(S)} \\ A_L^{(S)} \end{bmatrix} = f^{(BA)} F^N \begin{bmatrix} A_S^{(S)} \\ A_L^{(S)} \end{bmatrix}, \tag{5}$$

where N is the number of cells, $A_J^{(j)}$ is an amplitude of the displacement of the mode J in the layer j (D for the detector layer and S for substrate). Further,

$$T=M^{(B)}\Phi^{(B)}\left[M^{(B)}\right]^{-1}M^{(A)}\phi^{(A)}\left[M^{(A)}\right]^{-1}, \tag{6}$$

$$F=\Phi^{(B)}f^{(AB)}\Phi^{(A)}f^{(BA)}, \tag{7}$$

$$M^{(j)}=\begin{bmatrix} e_{Sx}^{(j)} & e_{Lx}^{(j)} \\ e_{Sz}^{(j)} & e_{Lz}^{(j)} \end{bmatrix}, \tag{8}$$

$$\Phi^{(j)}=\begin{bmatrix} e^{i\zeta_{S+}^{(j)}H_j} & 0 \\ 0 & e^{i\zeta_{L+}^{(j)}H_j} \end{bmatrix}, \tag{9}$$

$$f^{(jk)}=\left[M^{(k)}\right]^{-1}M^{(j)}=\frac{1}{(e_L^{(k)},\ e_S^{(k)})}\begin{bmatrix} (e_L^{(k)},\ e_S^{(j)}) & (e_L^{(k)},\ e_L^{(j)}) \\ -(e_S^{(k)},\ e_S^{(j)}) & -(e_S^{(k)},\ e_L^{(j)}) \end{bmatrix}, \tag{10}$$

where $e_J^{(j)}$ is a polarization vector of the mode J propagating to the positive direction of z-axis in the layer j (= A or B). The quantities $e_{Jx}^{(j)}$, $e_{Jz}^{(j)}$ are x-component and z-component of $e_J^{(j)}$, respectively. The polarization vector is normalized as $|e_J^{(j)}|=1$. $\Phi^{(j)}$ represents phase change for phonons to get through the layer j. In Eq.(10), $(e_K^{(k)},\ e_J^{(j)})$ means y-component of the outer product $e_K^{(k)} \times e_J^{(j)}$. The 2×2 matrix T is a transfer matrix without any reflections. The matrices $f^{(BA)}$ and $f^{(AB)}$ defined by Eq.(10) are matrices of amplitude transmission coefficients. They are calculated from the polarization vectors of the two phonon modes, and they are real.

When the resonant condition given by Eq.(4) is satisfied, the matrix F defined by Eq.(7) becomes

$$F=e^{i\theta}R \tag{11a}$$

$$R=\begin{bmatrix} f_{11}^{(AB)}f_{11}^{(BA)}+e^{i\phi}f_{12}^{(AB)}f_{21}^{(BA)} & f_{11}^{(AB)}f_{12}^{(BA)}+e^{i\phi}f_{12}^{(AB)}f_{22}^{(BA)} \\ e^{-i\phi}f_{21}^{(AB)}f_{11}^{(BA)}+f_{22}^{(AB)}f_{21}^{(BA)} & e^{-i\phi}f_{21}^{(AB)}f_{12}^{(BA)}+f_{22}^{(AB)}f_{22}^{(BA)} \end{bmatrix}, \tag{11b}$$

where

$$\phi\equiv(\zeta_{S+}^{(B)}-\zeta_{L+}^{(B)})H_B=(\zeta_{L+}^{(A)}-\zeta_{S+}^{(A)})H_A+2\pi m, \tag{12}$$

and m is an integer and θ is defined by Eq.(4). (cf. the appendix for derivation of the expression of F). At the resonant condition, a phase factor $e^{i\theta}$ appears in the expression of the matrix F. This factor is neglectable, because

$|A_J^{(j)}|^2$ has physical meaning.With the matrix R, we get a relation of the amplitudes between *adjacent n*-th and $(n+1)$-th *cells* as follows:

$$\begin{bmatrix} A_S^{<n+1>} \\ A_L^{<n+1>} \end{bmatrix} = R \begin{bmatrix} A_S^{<n>} \\ A_L^{<n>} \end{bmatrix}. \qquad (13)$$

This is a set of simultaneous finite difference equations. The matrix R has the following features:

$$\det[R] = 1, \qquad (14a)$$
$$\mathrm{tr}[R] = 2 - 2(1-\cos\psi) f_{12}^{(AB)} f_{21}^{(BA)}, \qquad (14b)$$
$$R_{22} = R_{11}^{*}. \qquad (14c)$$

The value of Eq.(14b) is real. With an initial condition:

$$A_S^{<0>} = I_0, \qquad (15a)$$
$$A_L^{<0>} = 0; \qquad (15b)$$

Eq.(13) has the following solution:

$$|A_S^{<n>}|^2 = |I_0|^2 \cos^2(\phi n) + \varepsilon, \qquad (16a)$$
$$\gamma |A_L^{<n>}|^2 = |I_0|^2 \sin^2(\phi n), \qquad (16b)$$
$$\varepsilon \equiv \frac{|R_{11}-R_{22}|^2}{4\sin^2\phi} |I_0|^2 \sin^2(\phi n),$$
$$\gamma \equiv \frac{\sin^2\phi}{|R_{21}|^2},$$

where ϕ is a real number and defined as follows:

$$\cos\phi = \frac{1}{2}\mathrm{tr}[R] = 1 - (1-\cos\psi) f_{12}^{(AB)} f_{21}^{(BA)}. \qquad (17)$$

With this solution, we can give similar oscillations shown in Fig.6. A period of the oscillations is given by π/ϕ as 13.5. This value is very close to $\Delta N = 13.6$ given by the precise numerical calculations. The second term ε of Eq.(16a) is neglectable according to numerical calculations. Hence, Eq.(16a) and Eq.(16b) can represent the energy conservation approximately for the transmitted phonons and γ is ratio of acoustic impedances of mode L to mode S, i.e.

$$|A_S^{<n>}|^2 + \gamma |A_L^{<n>}|^2 \simeq |I_0|^2 \qquad (18)$$

At frequency off the resonant condition, the amplitudes of the oscillations decrease. In the phonon stop bands, reflections of phonons appear and the oscillations of transmission rates are

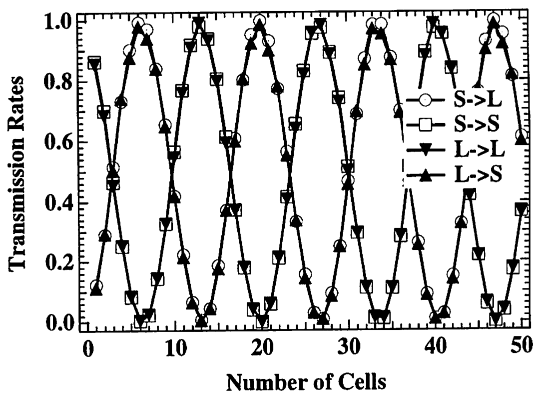

Fig.6. Oscillations of transmission rates calculated from 4 × 4 transfer matrix. Phonon incident angle in the substrate is 20 degrees for S-mode and 34.2 degrees for L-mode. The frequency is 117.6 GHz. The period of oscillations against number of cells is ΔN=13.6.

suppressed.

5. Summary and Conclusion

The structure factors can reveal the frequency dependence, the angular dependence of the transmission rates with mode conversion, and the reflection rates with/without mode conversions. However, they cannot reveal the oscillations against number of cells explicitly. They can only tell the frequencies at which the oscillations should occur. At the frequencies, the structure factor for transmission with mode conversions has the peaks, and the structure factors for reflections prohibit all kinds of reflections. To explain the oscillation, we need the 2×2 matrix F rather than the structure factors.

We consider the SL with the (111) interfaces, but the oscillations of the transmission rates exist also in a SL with the (001) interfaces and in a SL with isotropic layers.[3]

We get the resonant condition defined by Eq.(4) in the SL. The amplitude transmission coefficients can be got from the polarization vectors of the two modes. If the frequency and the incident angle satisfy the resonant condition , we can expect the oscillations of the transmission rates without any reflection. The oscillations are explained by the 2×2 matrix F given by Eq.(11). The resonant condition depends on the frequency, the incident angle, and the structure of the unit cell. These dependences can be explained from the structure factor with mode conversion defined by Eq.(3).

Even if the resonant condition is satisfied, there is a case that the transmission rates do not oscillate. In this case, the reflections are not neglectable. Therefore, the resonant condition given by Eq.(4) cannot always tell the frequency at which the oscillations should occur explicitly. Hence, we have to be careful also about the reflection rates when we discuss the oscillations. Anti-crossing of dispersion relation is also important to find oscillations. However, it does not always tell the oscillations. (For instance, see the dispersion relation in Fig.4(c) at frequencies about 90 GHz and 170 GHz.) All of oscillations against number of cells are not explained in the present paper. For instance, there is another oscillation between T_{SS} and R_{SL} if the frequency is 35 GHz and the S-mode incidence angle is 20 degrees. These features are still left for futher study.

When we would design SL's, we should be careful of the number of cells because of the oscillations for oblique phonon incidences. The oscillations also occur in a SL with anisotropic layers as well as in a SL with isotropic layers.[3] However, we should study the anisotropic effect (e.g. the phonon focusing and the conical refraction).

Appendix

A1. Components of the Matrix F

The component expression of the matrix F at the resonant condition is derived in this appendix . The definition of F is as follows:

$$F=\Phi^{(B)}f^{(AB)}\Phi^{(A)}f^{(BA)}. \qquad (A1)$$

To get the simplified form of the matrix F, we first calculate commutation relations:

$$C\equiv f^{(AB)}\Phi^{(A)}-\Phi^{(A)}f^{(AB)}, \qquad (A2)$$
$$C'\equiv \Phi^{(B)}f^{(AB)}-f^{(AB)}\Phi^{(B)}. \qquad (A3)$$

Making use of these matrices, we can express F in two ways as follows:

$$F=\Phi^{(B)}\Phi^{(A)}+G, \qquad (A4)$$
$$F=f^{(AB)}\Phi^{(B)}\Phi^{(A)}f^{(BA)}+G', \qquad (A5)$$

where

$$G\equiv \Phi^{(B)}Cf^{(BA)}, \qquad (A6)$$
$$G'\equiv C'\Phi^{(A)}f^{(BA)}. \qquad (A7)$$

At the resonant condition, the matrices $\Phi^{(j)}$'s have the following feature:

$$\Phi^{(B)}\Phi^{(A)}=e^{i\theta}\mathbf{1}, \qquad (A8)$$

where θ represents the phase change of phonons to get through a unit cell, and defined as follows:

$$\theta\equiv\zeta_{S+}^{(A)}H_A+\zeta_{S+}^{(B)}H_B=\zeta_{L+}^{(A)}H_A+\zeta_{L+}^{(B)}H_B+2\pi m, \quad (A9)$$

where m is an integer. (This is the resonant condition.) With this condition, Eq.(A4) and Eq.(A5) become

$$F=e^{i\theta}\mathbf{1}+G, \qquad (A10)$$
$$F=e^{i\theta}\mathbf{1}+G'. \qquad (A11)$$

Because Eq.(A10) and Eq.(A11) are identical to each other, we can put $G=G'$. Therefore, $G=\frac{1}{2}$ (Eq.(A6) + Eq.(A7)) . After some calculations by making use of the relation $f^{(AB)}f^{(BA)}=\mathbf{1}$, we can get the following expression:

$$G=-e^{i\theta}\mathbf{1}+\begin{bmatrix} e^{i\theta}f_{11}^{(AB)} & af_{12}^{(AB)} \\ bf_{21}^{(AB)} & e^{i\theta}f_{22}^{(AB)} \end{bmatrix}f^{(BA)}, \qquad (A12)$$

where

$$a\equiv e^{i(\zeta_{S+}^{(B)}H_B+\zeta_{L+}^{(A)}H_A)}, \qquad (A13)$$
$$b\equiv e^{i(\zeta_{L+}^{(B)}H_B+\zeta_{S+}^{(A)}H_A)}. \qquad (A14)$$

Using of Eq.(A9) again for Eqs.(A13) and (A14), we get the following equation:

$$\psi\equiv(\zeta_{S+}^{(B)}-\zeta_{L+}^{(B)})H_B=(\zeta_{L+}^{(A)}-\zeta_{S+}^{(A)})H_A+2\pi m. \quad (A15)$$

where m is an integer. With this equation, Eq.(A12) becomes

$$G=-e^{i\theta}\mathbf{1}+e^{i\theta}\begin{bmatrix} f_{11}^{(AB)} & e^{i\phi}f_{12}^{(AB)} \\ e^{-i\phi}f_{21}^{(AB)} & f_{22}^{(AB)} \end{bmatrix} f^{(BA)}. \qquad (A16)$$

Substituting Eq.(A16) into Eq.(A10), we get the matrix F as follows:

$$F=e^{i\theta}\begin{bmatrix} f_{11}^{(AB)}f_{11}^{(BA)}+e^{i\phi}f_{12}^{(AB)}f_{21}^{(BA)} & f_{11}^{(AB)}f_{12}^{(BA)}+e^{i\phi}f_{12}^{(AB)}f_{22}^{(BA)} \\ e^{-i\phi}f_{21}^{(AB)}f_{11}^{(BA)}+f_{22}^{(AB)}f_{21}^{(BA)} & e^{-i\phi}f_{21}^{(AB)}f_{12}^{(BA)}+f_{22}^{(AB)}f_{22}^{(BA)} \end{bmatrix}. \qquad (A17)$$

A2. Equations of Howie and Whelan

To explain the Pendellösung effect in a SL, we get a set of simultaneous differential equations. First we have to remove the phase factor $e^{i\theta}$ from the matrix F, because it does not contribute the values for $|A_S|^2$ and $|A_L|^2$. Making use of. $f^{(AB)}f^{(BA)}=1$ again, we can define the following matrix $Q=\frac{1}{e^{i\theta}}G$:

$$Q=\frac{1}{e^{i\theta}}F-\mathbf{1}=\begin{bmatrix} e^{i\phi}-1 & 0 \\ 0 & e^{-i\phi}-1 \end{bmatrix} \begin{bmatrix} f_{12}^{(AB)}f_{21}^{(BA)} & f_{12}^{(AB)}f_{22}^{(BA)} \\ f_{21}^{(AB)}f_{11}^{(BA)} & f_{21}^{(AB)}f_{12}^{(BA)} \end{bmatrix}. \qquad (A18)$$

If $|\phi|\ll 1$, we can make an approximation $e^{\pm i\phi}\cong 1\pm i\phi$, and get Q as follows:

$$Q=\begin{bmatrix} i\phi f_{12}^{(AB)}f_{21}^{(BA)} & i\phi f_{12}^{(AB)}f_{22}^{(BA)} \\ -i\phi f_{21}^{(AB)}f_{11}^{(BA)} & -i\phi f_{21}^{(AB)}f_{12}^{(BA)} \end{bmatrix}. \qquad (A19)$$

If there is an integer m that satisfies a relation $|\phi-2\pi m|\cong 0$, we can also make a similar approximation $e^{\pm i\phi}\cong 1\pm i(\phi-2\pi m)$. With this matrix Q, we can derive a set of simultaneous differential equations for the amplitudes of displacements as follows:

$$\frac{d}{dz}\begin{bmatrix} A_S \\ A_L \end{bmatrix}=\frac{1}{D}Q\begin{bmatrix} A_S \\ A_L \end{bmatrix}, \qquad (A20)$$

where $D=H_A+H_B$, i.e. the thickness of a unit cell. Because $\mathrm{tr}[Q]=0$ and $\det[Q]$ is a positive real number, Eq.(A20) has a solution that $|A_S|^2$ and $|A_L|^2$ oscillate, and is identical to the equations (4a) and (4b) of Howie and Whelan.[4]

If there is an in teger m that satisfies $\phi=2\pi m$, then $Q=\mathbf{0}$, $G=\mathbf{0}$, and $F=e^{i\theta}\mathbf{1}$. Hence, there is a possibility that incident phonons could get through the SL without any reflections and without any mode conversions from the substrate to the detector layer.

References

[1] S. Tamura, D.C. Hurley, and J.P. Wolfe, Phys. Rev. **B 38**, 1427(1988).

[2] S. Tamura, Phys. Rev. **B 43**, 12646(1991).

[3] H. Kato, H. J. Maris, S. Tamura, Phys. Rev. **B53**, 7884(1996); Physica **B 219 & 220**, 696(1996).

[4] A. Howie and M.J. Whelan, Proc. Roy. Soc. **A263**, 217(1961).

[5] N. Kato, Acta Cryst. **16**, 276(1963); **14**, 526(1961). B.E. Warren, *X-ray Diffraction*, p339, (Addson-Wesley,1969).

[6] H. Kato, J. Acoust. Soc. Am., to be published. For basic formulations: B. A. Auld, *Acoustic Fields and Waves in Solids*, 2nd Ed. vol. I and II (Robert E. Krieger,1990).