Resonant transmission of acoustic phonons from a superlattice into uniform materials

Hatsuyoshi KATO*

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ABSTRACT

Resonant transmission appears for acoustic phonons that propagate from a finite-size superlattice into uniform materials even if the large mismatch of their acoustic impedances exists. Frequencies at which the transmission rate has maximum values coincide with those of surface vibrational modes in the finite-size superlattice. To characterize this phenomenon, an effective acoustic impedance and an effective reflection coefficient are introduced.

Ⅰ. INTRODUCTION

Acoustic vibrational fields have been studied extensively in multilayered structures whose sizes are infinite, semi-infinite, and finite. In the infinite structures, displacements of the acoustic field have quasi-periodic properties at passing frequencies of dispersion relations. However, at stopping band frequencies, the displacements attenuate exponentially [1]. Semi-infinite multilayers have surface waves at stress-free surfaces [2]. Experimentally, it has been reported to detect surface mode at the superlattice (SL) surface with the pump and probe method [3].

Meanwhile, resonant transmissions of phonons are reported for triple-superlattice structures and a bulk material sandwiched by two superlattices [4-6]. According to these analyses, the resonant frequencies are at acute frequency regions in stop bands of the SL dispersion relation.

Recently, the wave front imaging experiment has been developed for observing the shape of a vibratinal wave front on crystalline solid surfaces [7]. A focused acoustic beam from a ultrasonic transducer excites a small spot on one face of a solid that is immersed in a liquid bath. An identical transducer is focused on the opposite face and receives the acoustic wave transmitted through the crystal. The receiving transducer is raster-scanned, and gets a spatial distribution of the transmitted flux that appears on the solid surface and gives data of the expanding wave front striking the crystal surface.

If this wave front imaging experiment is applied to detecting the mode converted transmissions in periodic and aperiodic SL's in previous papers [8,9], we can expect the detection of the mode conversions. We try to propose experimental methods to detect these mode converted transmissions (see appendix).

Ⅱ. TRANSMISSION RATE AND PHASE TIME

In the present paper, we study a finite-size SL as in Fig. 1 with configuration that one surface of the SL is connected with uniform material. In this system, a resonant transmission of acoustic phonons into uniform material from the SL is investigated. In the SL, two kinds of layers (A and B) are alternately stacked. The substrate of the SL is assumed as the same substance as in the layer B for simplicity. From this substrate, acoustic phonons are incident normally to the interfaces of the SL layers. This resonance is characterized by an effective acoustic impedance [10].

Making use of the transfer matrix method, we express the relation of phonon amplitudes between the substrate and a boundary connected with the SL and the uniform material :

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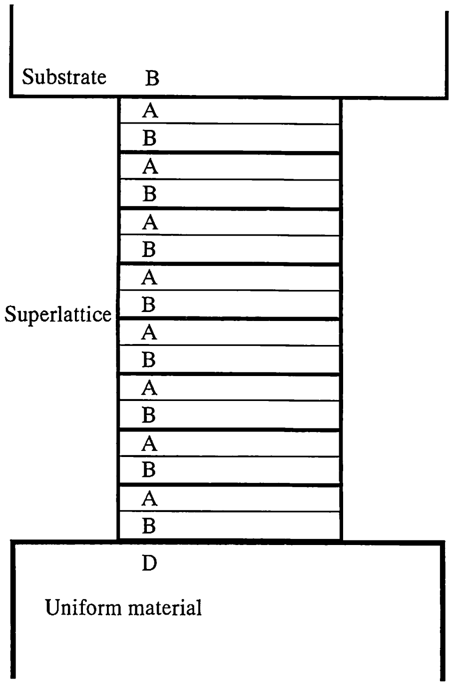

FIG. 1. Configuration of the system we consider. One bilayer is made of materials A and B. The substate is assumed to be of material B which is the same as one in the bilayer. The number of bilayers is expressed by N in text, and it is eight in this figure. Acoustic impedances are Z_A and Z_B for the materials A and B, respectively. For the uniform material, the acoustic impedance is expressed by Z_D.

$$\begin{bmatrix} a_N \\ \kappa a_N \end{bmatrix} = G^N \begin{bmatrix} a_0 \\ b_0 \end{bmatrix}, \tag{1}$$

where N is the number of the bilayers AB, a_0 and b_0 are incident amplitude and reflected amplitude in the substrate, respectively. The quantity a_N is the amplitude of the acoustic phonons reached at the boundary of the SL from the substrate. The value κ expresses the amplitude reflection coefficient for phonons to be reflected from the boundary back to the SL. The 2 × 2 matrix G is the transfer matrix for one bilayer, and each element of the producted matrix G^N becomes

$$(G^N)_{11} = \cos N\gamma + is(\gamma)g(\alpha,\beta), \tag{2}$$

$$(G^N)_{22} = \cos N\gamma - is(\gamma)g(\alpha,\beta), \tag{3}$$

$$(G^N)_{12} = ie^{i\beta}s(\gamma)\frac{1}{2}(Z_A/Z_B - Z_B/Z_A)\sin\alpha, \tag{4}$$

$$(G^N)_{21} = -ie^{-i\beta}s(\gamma)\frac{1}{2}(Z_A/Z_B - Z_B/Z_A)\sin\alpha, \tag{5}$$

where α and β are real and they are the products of wave numbers of the acoustic phonons and thicknesses of the layer A and of the layer B, respectively ; and

$$g(\alpha,\beta) = \cos\alpha\sin\beta + \frac{1}{2}(Z_A/Z_B + Z_B/Z_A)\sin\alpha\cos\beta. \tag{6}$$

Acoustic impedances are expressed by the values Z_A for the layer A and by the value Z_B for the layer B. Function $s(\gamma)$ is given by the following definitions :

$$s(\gamma) = \frac{\sin N\gamma}{\sin\gamma}, \tag{7}$$

where γ is given by the eigenvalues of the matrix G and expressed as

$$\cos\gamma = \cos\alpha\cos\beta - \frac{1}{2}(Z_A/Z_B + Z_B/Z_A)\sin\alpha\sin\beta. \tag{8}$$

Because of the features of the eigenvalues, $\cos\gamma$ is equal to one half of tr $[G]$. This value γ becomes complex number when the right hand side of Eq. (8) requires $|\cos\gamma| > 1$.

From the above expressions, we get amplitude reflection coefficient which is a ratio of the reflected amplitude to the incident amplitude in the substrate as follows :

$$r = \frac{b_0}{a_0} = \frac{\kappa(G^N)_{11} - (G^N)_{21}}{(G^N)_{22} - \kappa(G^N)_{12}}. \tag{9}$$

Therefore, the reflection rate of the SL becomes $R = |r|^2$. Amplitude transmission coefficient that expresses the fraction of amplitude getting through the SL to the boundary is

$$t = \frac{a_N}{a_0} = (G^N)_{11} + r(G^N)_{12} = \frac{1}{(G^N)_{22} - \kappa(G^N)_{12}}, \tag{10}$$

where we use det $[G^N] = 1$.

Amplitude a_D of the phonons penetrating into the uniform material from the boundary of the SL has a relation as follows [11] :

$$\begin{bmatrix} a_D \\ 0 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1+Z_B/Z_D & 1-Z_B/Z_D \\ 1-Z_B/Z_D & 1+Z_B/Z_D \end{bmatrix}\begin{bmatrix} a_N \\ \kappa a_N \end{bmatrix}, \tag{11}$$

where Z_D is acoustic impedance of the uniform material. In this expression, we assume that no reflections occur from the uniform material to the boundary. The above relation gives amplitude reflection coefficient at the boundary as

$$\kappa = \frac{1-Z_D/Z_B}{1+Z_D/Z_B}. \tag{12}$$

This expression implies κ is real because the acoustic impedances Z_D and Z_B are also real. From this relation, the ratio of the two impedances becomes

$$\frac{Z_D}{Z_B} = \frac{1-\kappa}{1+\kappa}. \tag{13}$$

Further, amplitude transmission coefficients for the penetration to the uniform material from the boundary and from the substrate are

$$t_{DN} = \frac{a_D}{a_N} = 1+\kappa, \tag{14}$$

$$t_{D0} = \frac{a_D}{a_0} = (1+\kappa)t. \tag{15}$$

Knowing these expressions, we get the transmission rate of the phonons from the substrate to the uniform material as follows :

$$T = \frac{Z_D}{Z_B}\left|\frac{a_D}{a_0}\right|^2 = (1-\kappa^2)|t|^2, \tag{16}$$

where we use Eqs. (13) and (15).

The phase time for transmission [12-14] that expresses the time for phonons to get through from the substrate to the uniform material is given by Eq. (15) as $\tau = d\Theta/d\omega$, where ω is angular frequency of the phonons, and

$$\Theta = -\arg[(G^N)_{22} - \kappa(G^N)_{12}] \tag{17}$$

is the phase expressed as $t_{D0} = |t_{D0}|e^{i\Theta}$ from Eqs. (10) and (15).

Ⅲ. NUMERICAL RESULTS

We show some results of numerical calculations for SL's that have the layers of GaAs and AlAs. The uniform material is assumed as distilled water. The number N of the bilayer AB in the SL is eight, and each layer A or B has fifteen monolayers with (100) interfaces. Thickness of the monolayer is 2.83 Å . Acoustic impedances for longitudinal (L) mode phonons are 25.2 for GaAs, 21.2 for AlAs, and 1.48 for the distilled water in unit of 10^5 g ·cm^{-2} · s^{-1}.

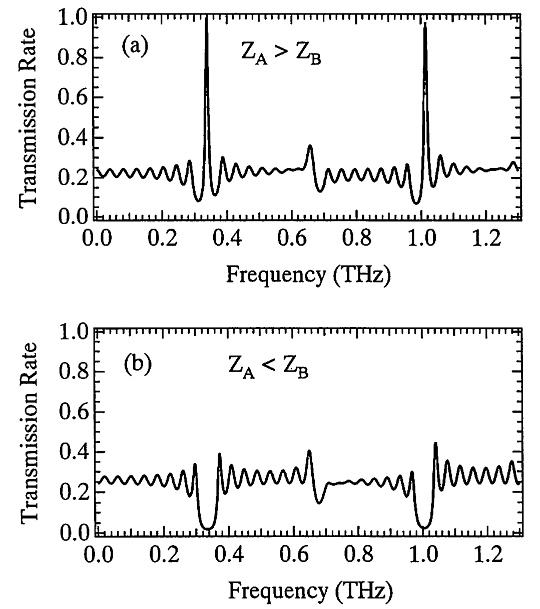

FIG. 2. The layers of the SL are consist of GaAs and AlAs. The uniform material is assumed as distilled water. The number of the bilayer AB in the SL is 8, and each layer has 15 monolayers with (100) interfaces. (a) The transmission rate of the *L* mode phonons into distilled water when the layer A is of GaAs and the layer B is of AlAs. (b) The layer A is of AlAs and the layer B is of GaAs.

When the layer A is of GaAs and the layer B is of AlAs, the acoustic impedances satisfy an inequality $Z_A > Z_B$. In this case, the transmission rate T calculated from Eq. (16) for the L mode phonons has sharp peaks at certain frequencies as in Fig. 2 (a). For instance, one of the frequencies is ν_G = 338 GHz, when the following two relations are satisfied simultaneously ;

$$\sin\beta = \sin\alpha > 0, \quad \cos\beta = -\cos\alpha > 0.$$

In case of the finite-size SL we now consider, vibrational amplitude of the displacement at the boundary becomes $(1+\kappa)\,a_N$ from Eq.(1). The magnitude of the displacement becomes 1.87 × 2.03 (κ = 0.870) greater at the frequency ν_G. Therefore, this localized vibration at the boundary makes the L mode phonons in the SL penetrate efficiently into the distilled water. (The amplitude is 2 × 3.83 ($\kappa = 1$) times greater than the incident amplitude in the substrate without the distilled water at frequency ν_G.) These resonant frequencies are in the stop bands which appear in infinite-size SL's. Except these resonant frequencies, the reflection rate R is near the value κ^2 = 0.757 and the transmission rate is $T = 1 - \kappa^2 = 0.243$, which are for the bulk AlAs and the distilled water.

When the materials of layers A and B are interchanged, i.e., layer A is of AlAs and layer B is of GaAs, then $Z_A < Z_B$, and the transmission rate becomes zero at the above resonant frequencies as in Fig. 2 (b). In this case, the influence of the stop bands appears and the incident L mode phonons are reflected by the Bragg reflection [11, 12].

As in Fig. 3, the height of the resonant peak at ν_G = 338 GHz depends on N which is the number of bilayers. When $Z_A > Z_B$, then the transmission rate have a peak at N = 8. This is why we first consider the SL with eight bilayers. On the other hand, no peaks exist when $Z_A < Z_B$. This phenomenon is explained to some extent by the unitarity of the transfer matrix G. In Fig. 4, we show (a) dispersion relation for the infinite size SL, (b) absolute values of det $[G]$ and absolute values of $\frac{1}{2}$tr $[G]$, and (c) absolute values of the eigenvalues of G. In frequency regions of stop bands, $|\frac{1}{2}\mathrm{tr}\,[G]| > 1$. Further, $|\det[G]| = 1$, but the absolute values of eigenvalues of G are not unity. This implies that the transfer matrix G is not unitary in the stop band frequencies [9]. Therefore, the total energy of propagating and reflected phonons expressed as $|a_n|^2 + |b_n|^2$ in the n-th bilayer does not conserve unlike the energy of phonons in the passing bands. An instance of this effect is displayed in Fig. 3. The transmission rate has a peak in case of $Z_A > Z_B$. If $Z_A < Z_B$, then the transmission rate decreases as N increases.

Phase time of the transmission is displayed in Fig. 5. Figure 5 (a) is with the same condition of Fig. 2 (a). We see that it takes long time for the phonons at the resonance to get through the SL into the distilled water in case of $Z_A > Z_B$. This result implies that the incident L mode phonons are captured by the boundary vibrational mode and penetrate gradually into the water. Except resonant frequencies, the phase time is near the value 13.2 ps that is time when we neglect the effects of interfaces in the SL. If $Z_A < Z_B$, then the time at the resonance is not long as in Fig. 5 (b). The latter time must be considered as a time of one way travel of the phonons from the substrate toward the boundary for the Bragg reflection.

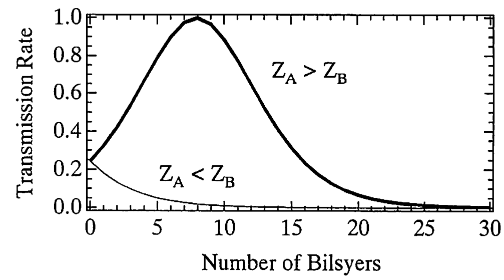

FIG. 3. Transmission rate against the number of bilayers. Frequency is set as ν_G (see text).

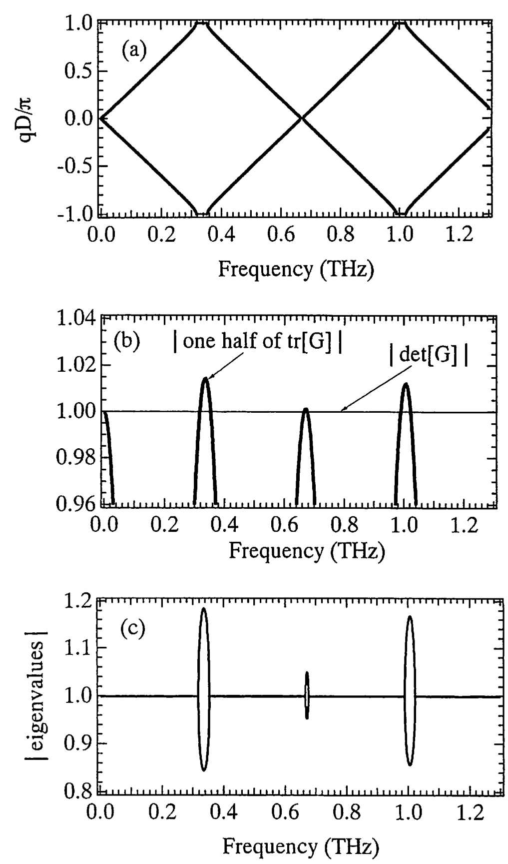

FIG. 4. Characteristics of the transfer matrix G : (a) dispersion relation of L mode phonons in infinite size SL, (b) absolute values of $\frac{1}{2}$tr$[G]$and det$[G]$, and (c) absolute values of eigenvalues of G. The quantity q is superlattice wave number and D is the thickness of one bilayer.

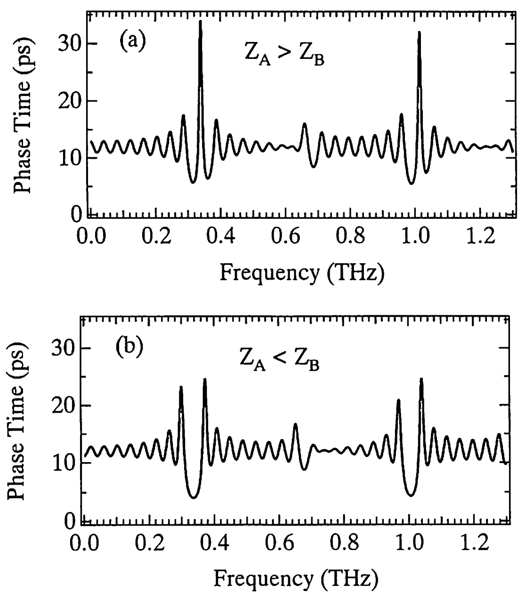

FIG. 5. Phase time of transmission for L mode phonons to propagate from the substrate into the uniform material : (a) when the layer A is of GaAs and the layer B is AlAs ($Z_A > Z_B$), (b) when the layer A is of AlAs and the layer B is GaAs ($Z_A < Z$).

Ⅳ. EFFECTIVE IMPEDANCE

The amplitude reflection coefficient κ satisfies an inequality $1 \geq \kappa > -1$. If Z_D is zero, then $\kappa = 1$ and this case expresses the boundary is exposured to vacuum because of Eq.(12). When Z_D equals to Z_B, then κ is zero and expresses the uniform material has the same acoustic impedance with that of the layer B. Further, if $Z_D < Z_B$, then κ is positive. On the other hand, if $Z_D > Z_B$, then κ is negative.

In Fig.6(a), the transmission rate is plotted when the layer A is of GaAs and the layer B is of AlAs in the SL, i.e., $Z_A > Z_B$. The transverse axis is for the impedance ratio Z_D/Z_B on a log scale. The transmission rate is labeled "SL" for a curve of the superlattice. The transmission rate labeled "bulk" is for a case of bulk material with the same substance as the layer B which is of AlAs in the present case. Frequency is ν_G = 338 GHz and the other condition is the same as in Fig.2(a). When the uniform material is distilled water, he ratio is 6.98 × 10^{-2} and it has the value that is optimal for the resonance transmission. The other liquid which is appropriate to the resonance includes toluene and benzene, and their impedance ratios are 5.40 × $10^{-2}$and 5.48 × 10^{-2}, respectively. In Eq.(16), the factor $(1-\kappa^2)$ is the transmission rate for bulk materials and has one peak only at $\kappa = 0$ $(Z_D = Z_B)$. The other factor $|t|^2$ expresses an effect of the SL to shift the peak of the transmission rate to lower impedance ratios as in Fig.6(b). The factor $|t|^2$ also has one peak only at $\kappa = \mathrm{Re}\{c\}$ where $c = (G^N)_{22}/(G^N)_{12}$. If this value c is positive at the resonant condition, the peak of $(1-\kappa^2)$ shifts to lower impedance ratios and makes the peak for the SL. In case of Fig.6, Re$\{c\}$ = 1.15.

The peak of the transmission rate for the SL also shifts to very high acoustic impedance regions when $Z_A < Z_B$ as in Fig.7. In this case, the layer A is of AlAs and the layer B is of GaAs. The resonant frequency is 333 GHz, the value κ = -0.873, and the value Re$\{c\}$ = −1.14. The peak of the transmission rate of the SL with AlAs/GaAs is at an impedance which is greater than the diamond acoustic impedance for L-mode phonons. Therefore, we cannot expect to realize this resonance. However, it is possible to make phonons transmit resonantly from a SL or multilayered structure with polyethylene and polystyrene bilayers to some solids like Si, AlAs, and GaAs. In this case, the optimal number of bilayers is three, and the resonant peak is at 6.66 times of the polystyrene impedance. Acoustic impedances for L-mode phonons of polyethylene, polystyrene, and Si are 1.76, 2.48, and 19.6, respectively in unit of 10^5 g· $\mathrm{cm}^{-2} \cdot \mathrm{s}^{-1}$. The optimal uniform material is Si for the resonance in this plastic multilayered structure.

In both cases of Fig. 6 and Fig. 7, the transmission rates of the two SL's have similar curves as the transmission rate for the bulk material. This implies that effective impedance of a SL is definable at the resonance.

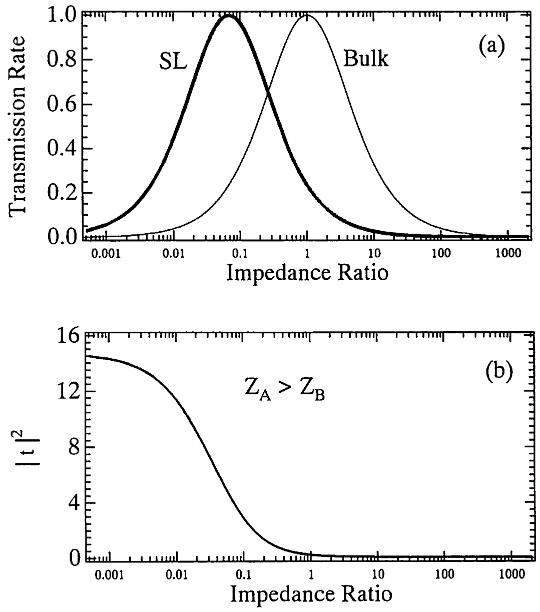

FIG. 6. (a) Transmission rates for both of the SL and the bulk materials against ratios of the impedance of the uniform material to that of the layer B, when the layer A is of GaAs and the layer B is of AlAs ($Z_A > Z_B$). Frequency is ν_G. The peak for the bulk materials is shifted to lower impedances by virtue of the SL. In case that the uniform material is distilled water, the impedance ratio is 6.98×10^{-2}. (b) The effect of the SL is expressed by $|t|^2$.

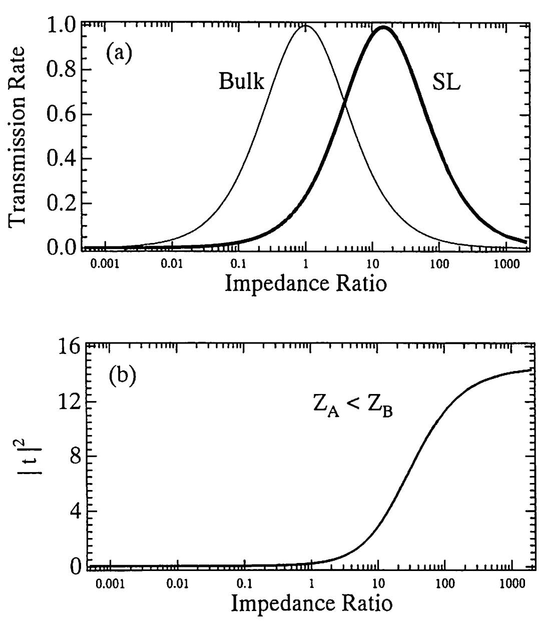

FiG. 7. When the layer A is of AlAs and the layer B is of GaAs ($Z_A < Z_B$) : (a) Transmission rates for both of the SL and the bulk materials against the impedance ratios (see test). (b) Also in this case, the effect of the SL is expressed by $|t|^2$

V. CONCLUSION

Exact calculation to obtain the value of κ for the peak of the transmission rate T gives a cubic equation and it is modified as follows with expressing the value by κ_e :

$$\kappa_e = \frac{(3\kappa_e^2 - 1)\,\mathrm{Re}\{c\}}{2\kappa_e^2 - 1 + |c|^2}. \tag{18}$$

If we define the effective acoustic impedance Z_e of the SL by the peak of T, then Z_e is expressed using a relation of Eq.(13) as

$$Z_e = Z_B \frac{1-\kappa_e}{1+\kappa_e}. \tag{19}$$

If Z_D is equal to Z_e at the resonance, the transmission rate has the maximum value.

As a conclusion, the transmission rate of the SL's at the resonant frequency has the following expression:

$$T \simeq 1 - K^2, \tag{20}$$

where

$$K = \frac{1 - Z_D/Z_e}{1 + Z_D/Z_e} = \frac{\kappa - \kappa_e}{1 - \kappa\,\kappa_e} \tag{21}$$

is an effective reflection coefficient by using a relation of Eq.(12).

APENDIX:

Detection of mode conversions

We have studied the transmission and reflection of phonons incident on a periodic superlattice at oblique angle [8]. For frequencies in the vicinity of an anticrossing frequency in the superlattice phonon-dispersion relation, we have found that the wave energy oscillates back and forth between the different polarizations (longitudinal and transverse modes) as the wave propagates through the superlattice. To detect this phenomenon, we cut the superlattice and its substrate like wedges to make a hexahedron and immersed in liquid as in Fig. 8. If the longitudinal phonons are input from the substrate side, the vibration on the surface of the superlattice changes its directions from the right to the left in the figure. This is caused by the difference of the number of bilayers for phonons to get through the structure. If the phonons get through many interfaces, the mode conversion is enhanced. Therefore, the pressure in the liquid changes its magnitude above the receiver, which is expressed by open circles. The larger radius circle expresses the stronger pressure. We expect the pressure oscillates in the liquid above the receiver.

Anomalous mode-converted transmission of phonons is found based for the transmitted amplitudes of phonons propagating oblique to the layer interfaces of aperiodic superlattices [9]. With use of transfer matrix, unusual intermode oscillations are analyzed in phonon transmission in the aperiodic superlattices with variable thickness of bilyers. To detect this character, we cut the superlattice and its substrate like the same shape of the above discussion and immersed in liquid as in Fig. 9. If the longitudinal phonons are input from the substrate side, the pressure in the liquid changes its magnitude above the receiver, which is also expressed by open circles indicating its strength. In this case, we expect the pressure fades out from the right to the left in the liquid above the receiver.

In the above two examples, the pressure measured in the liquids has large magnitude when the structure of the superlattice and the incident angle of phonons are selected to satisfy the resonant transmission from the superlattices to liquids.

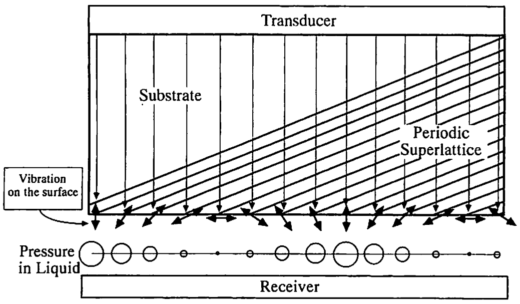

FIG. 8. Detection method of resonant-mode conversion in the periodic superlattice. Incident phonon mode from the transducer is longitudinal.

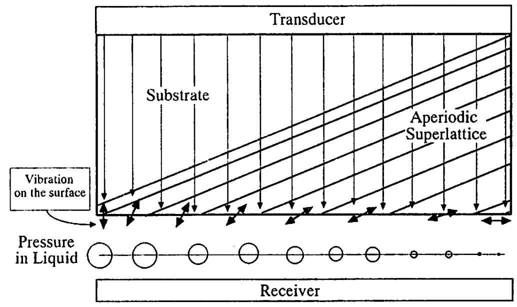

FIG. 9. Detection method of anomalous transmission in the aperiodic superlattice. Incident phonon mode from the transducer is longitudinal.

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