Transmission of acoustic energy in finite-size layered structures expressed by the effective acoustic impedance

Hatsuyoshi Kato*

Tomakomai National College of Technology, 443 Nishikioka, Tomakomai, 059-1275 Japan

(Received 19 November, 2001)

An effective acoustic impedance is defined for a finite-size layered structure. With this effective acoustic impedance, the transmission and the reflection of acoustic waves are expressed concisely. The frequencies of transmission enhancement are derived with the total number of layers in the layered structure, which include the resonant transmission frequencies. On the transmission enhancement conditions, the impedance at a surface of the layered structure coincide with the effective impedance. With considering energy fluxes, we relate surface vibrational magnitude closely to the impedance. Further, the effective acoustic impedance gives low energy flux at the transmission enhancement frequencies.

1 Introduction

In a previous study [1], the definition of effective acoustic impedance (EAI) is proposed. Further, the enhancement is shown is shown numerically for the transmission of acoustic waves. For this enhancement, the EAI of a layered structure (LS) must coincide with the acoustic impedance of liquid which is in contact with the LS.

In this paper, the EAI is defined with complex effective reflection coefficient (complex ERC) of the LS. Further, transmission rate and the reflection rate are expressed concisely with the EAI, even if the impedance matching is not established. When the impedance matching is set up, transmission rate becomes unity which is its maximum value. Resonant transmissions reported previously [1, 2] can be explained with this EAI matching.

To define the EAI, we need to treat complex reflection coefficients. Magnitudes of these complex coefficients have already been measured by experiments at solid-liquid interfaces with frequencies between 100 and 300 GHz. [3] In this study, the complex coefficients are assumed to originate from the loss in liquid. [4] Meanwhile, we need to consider complex coefficients in liquid and in solid if acoustic waves are travelling to directions both positive and negative.

The surface modes in Al/Ag superlattices have been studied experimentally with frequencies in the range from 110 to 670 GHz. [5] These results have also been compared with the predictions of a transfer-matrix theory of the localized modes. In this theory, they discuss the amplitude reflection coefficient at the surface and the dispersion relation which gives the frequency of a possible surface mode. However, more precise investigation shows that another condition is required. This condition relates the total number of layers in the finite-size LS closely to the surface mode frequencies. Further, the dispersion relation of surface modes is not realistic for the finite-size LS, if we neglect losses of acoustic energy in the LS system.

Recently, the low thermal conductance in LS has been discussed. [6, 7, 8] The conductance is reduced by a factor 10 compared to bulk semiconductors at room temperature. We show that this low thermal conductance is related to the low surface acoustic impedance which coincides with the EAI at the transmission enhancement frequencies. To derive this feature, we should be careful of stresses on the LS surface which cannot be removed, if we consider the finite-size LS in stationary states with monochromatic frequencies.

For the definition of the EAI, we consider liquid in contact with the LS. However, we do not have to persist in liquid. Materials in contact are allowed to be in any phases. To show this feature, we discuss acoustic waves at solid-solid interface and at solid-liquid interface (or solid-fluid interface) in the section 2. As a result, the liquid is treated as a working material to define the

* 助教授 電気電子工学科

EAI. The transmission and the reflection of acoustic waves in the LS are discussed in the section 3 with their numerical examples. We show that the surface vibration and energy fluxes are expressed with the EAI in the section 4. Further, we discuss energy flux incident from the liquid into the LS in the section 5. After these sections, related topics are discussed in the section 6. We conclude in the section 7. Fundamental concept is explained in appendices.

2 Interfaces

We discuss reflection coefficients and transmission coefficients at interfaces in this section. Relations of these coefficients are expressed in the same form at interfaces both of solid-solid and of solid-liquid. We use vectors and matrices defined in appendices A.1 and B.1. In the following discussions, the vectors $\Psi_A(x)$ and $\Psi_B(x)$ are translated from the vector $\Psi_S(x)$ defined in Eq. (52) for solid A and B, respectively, instead of solid S. The vector $\Psi_L(x)$ is defined for liquid in Eq. (71).

2.1 Solid-solid interface

If we assume that solid A is in a region $x < 0$ and solid B is in the other region $x > 0$, the condition for the continuity of displacement and stress is expressed with the vectors as $\Psi_A(-0) = \Psi_B(+0)$. [9] Its explicit expression with matrix elements is

$$\begin{bmatrix} a_B \\ b_B \end{bmatrix} = \frac{1}{2Z_B} \begin{bmatrix} Z_B + Z_A & Z_B - Z_A \\ Z_B - Z_A & Z_B + Z_A \end{bmatrix} \begin{bmatrix} a_A \\ b_A \end{bmatrix}. \tag{1}$$

Quantities a_A and b_A are displacement amplitudes, and Z_A is acoustic impedance of the solid A. Quantities a_B, b_B and Z_B are those for the solid B. We consider a case that the acoustic wave propagates from $x = -\infty$ to $x = 0$ in the solid A and it is reflected at the interface ($x = 0$) with the solid B by an amplitude reflection coefficient κ_{AB}. Further, we assume that there is only transmitted wave with an amplitude transmission coefficient τ_{AB} in the solid B. In this case, we can put that $b_A = \kappa_{AB}\ a_A$, $a_B = \tau_{AB}\ a_A$, and $b_B = 0$. Substituting these amplitudes in Eq. (1), we obtain the well-known coefficient expression with the acoustic impedances as

$$\kappa_{AB} = \frac{1 - Z_B/Z_A}{1 + Z_B/Z_A}, \tag{2}$$

$$\tau_{AB} = 1 + \kappa_{AB} = \frac{2}{1 + Z_B/Z_A}. \tag{3}$$

These expressions are derived for a solid-solid interface. However, we see that the same expressions are given for a solid-liquid interface in the section 2.2.

2.2 Solid-liquid interface

When a solid and a liquid are faced at $x = 0$, the continuity of solid displacement velocity and stress in the solid with acoustic field velocity and pressure in the liquid is expressed as $\Psi_S(-0) = \Psi_L(+0)$. [10] Its explicit expression with the matrix elements is

$$\begin{bmatrix} a_L \\ b_L \end{bmatrix} = \frac{-\omega/k_L}{2Z_L} \begin{bmatrix} Z_S + Z_L & -(Z_S - Z_L) \\ Z_S - Z_L & -(Z_S + Z_L) \end{bmatrix} \begin{bmatrix} a_S \\ b_S \end{bmatrix} \tag{4}$$

Quantities a_S and b_S are the displacement amplitudes, and Z_S is the acoustic impedance of solid S. Quantities a_L and b_L are amplitudes of velocity potential, and Z_L is acoustic impedance of the liquid L. From Eq. (4), we obtain a relation between $\xi_L = b_L/a_L$ and $\kappa = b_S/a_S$ as follows

$$\xi_L = \frac{(Z_S - Z_L) - (Z_S + Z_L)\kappa}{(Z_S + Z_L) - (Z_S - Z_L)\kappa}. \tag{5}$$

The quantity κ is an amplitude reflection coefficient in the solid S in contact with the liquid L, and ξ_L is that in the liquid. In general cases, the amplitudes have complex values. Therefore, the values of κ and ξ_L also become complex in Eq. (5).

In a case that the reflected waves do not exist in the liquid, i.e., $b_L = 0$ and $\xi_L = 0$, the quantity κ is given as

$$\kappa = \frac{1 - Z_L/Z_S}{1 + Z_L/Z_S}. \tag{6}$$

In this case, κ must be real, because both Z_L and Z_S are real. We define an amplitude transmission coefficient as a ratio of 'velocity' like $\tau = (ik_L\ a_L)/(-i\omega\ a_S)$. Using elements in Eq. (4), we obtain

$$\tau = \left(\frac{ik_L}{-i\omega}\right) \frac{\det[f^{(SL)}]}{(f^{(SL)})_{22}} = \frac{2}{1 + Z_L/Z_S} = 1 + \kappa, \tag{7}$$

and this value is also real. These coefficients, κ and τ, are identical to those of solids A-B discussed in the section 2.1, if we substitute the solid S for the solid A and the liquid L for the solid B. Therefore, we do not have to worry about material phases in contact with a LS in the section 3.

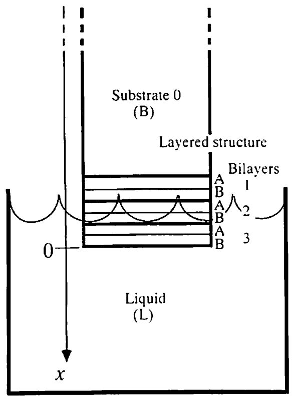

Figure 1: **An example of layered structure and liquid system. The space coordinate is x and has its origin at interface between the layered structure the liquid. The layered structure is in a region $x < 0$, and the liquid is in the other region $x > 0$. In the present paper, existence of the bottom of liquid bath is not considered, so that reflected waves from the bottom are neglected.**

3 Layered structure with liquid

3.1 Transmission and reflection

We discuss a system that a LS in contact with liquid as in Fig. 1. The LS is assumed to have a spatial period of unit bilayer AB and to have N bilayers. The LS extends in a region $x < 0$, and ends at $x = 0$. Its substrate is assume as the same solid as B. We assume that the contacted material with the LS is only a liquid L in the following discussions, because the expressions of amplitude reflection and transmission coefficients in the liquid have no differences from those in solid.

In the system $B(AB)_N$-L defined above, a relation of amplitudes in the solid B's between adjacent the n-th and the $(n{+}1)$-th bilayers becomes

$$\begin{bmatrix} a_{n+1} \\ b_{n+1} \end{bmatrix} = G \begin{bmatrix} a_n \\ b_n \end{bmatrix}, \tag{8}$$

where G is defined by Eq. (56). With iterations, we get a relation of amplitudes in the N-th layer and those in the substrate 0 as follows

$$\begin{bmatrix} a_N \\ \kappa\, a_N \end{bmatrix} = G^N \begin{bmatrix} a_0 \\ b_0 \end{bmatrix}, \tag{9}$$

where a_0 and b_0 are amplitudes in the substrate 0, and a_N is a amplitude in solid B of the N-th bilayer which is the end of the LS. The quantity κ is an amplitude reflection coefficient in the solid B faced with the liquid for the case of energy incidence from the LS substrate. It has well-known expression as follows

$$\kappa = \frac{1 - Z_L/Z_B}{1 + Z_L/Z_B}. \tag{10}$$

Because the acoustic impedances Z_B and Z_L are real, the reflection coefficient κ must be real. After some calculations, we obtain the following expressions for the elements of the matrix G^N.

$$(G^N)_{11} = \cosh N\zeta + is(\zeta)g(\alpha, \beta), \tag{11}$$

$$(G^N)_{12} = ie^{i\beta}s(\zeta)\ \frac{1}{2}(Z_A/Z_B - Z_B/Z_A)\sin\alpha, \tag{12}$$

$(G^N)_{22} = (G^N)^*_{11}$, $(G^N)_{21} = (G^N)^*_{12}$, where $\alpha = k_A\, d_A$, $\beta = k_B\, d_B$, and these are real. (Accurate definitions of these quantities in the above expressions are given in appendix A.1.) Further,

$$g(\alpha, \beta) = \cos\alpha\sin\beta + \frac{1}{2}(Z_A/Z_B + Z_B/Z_A)\sin\alpha\cos\beta, \tag{13}$$

$$\cosh\zeta = \cos\alpha\cos\beta - \frac{1}{2}(Z_A/Z_B + Z_B/Z_A)\sin\alpha\sin\beta, \tag{14}$$

$$\sinh^2\zeta = \frac{1}{4}(Z_A/Z_B - Z_B/Z_A)^2\sin^2\alpha - g^2(\alpha, \beta), \tag{15}$$

$$s(\zeta) = \frac{\sinh N\zeta}{\sinh\zeta}. \tag{16}$$

By a feature of the matrix G^N, we have $\cosh\zeta = \frac{1}{2}\mathrm{tr}[G]$. The right hand side of Eq. (14) defines the value ζ . If $\cosh\zeta > 1$, then ζ is real. If $\cosh\zeta < -1$, then ζ is complex number with a form of $\zeta = i\pi \times$ (odd integer) + (real). If $-1 \leq \cosh\zeta \leq 1$, then ζ is pure imaginary. The function $s(\zeta)$ depends also on N, and it gives real values for any values of ζ.

With the above expressions, the amplitude reflection coefficient $r = b_0/a_0$ in the substrate 0 is given as

$$r = \frac{\kappa\,(G^N)_{11} - (G^N)_{21}}{(G^N)_{22} - \kappa\,(G^N)_{12}}. \tag{17}$$

Because $(G^N)_{21} = (G^N)^*_{12}$ and $(G^N)_{11} = (G^N)^*_{22}$ are always true, Eq. (17) is reduced as follows

$$r = e^{i2\Theta}\ \frac{\kappa - \kappa_c}{1 - \kappa\,\kappa_c^*} = -e^{-i2\beta}\ \frac{\kappa/\kappa_c - 1}{1/\kappa_c^* - \kappa}, \tag{18}$$

where $\Theta = \arg[(G^N)_{11}]$, and we define $\kappa_c = (G^N)_{21}/(G^N)_{11}$ or explicitly with the matrix elements as follows

$$\kappa_c = \frac{-ie^{-i\beta}\frac{1}{2}(Z_A/Z_B - Z_B/Z_A)\ s(\zeta)\ \sin\alpha}{\cosh N\zeta + i\ s(\zeta)\ g(\alpha,\beta)}. \tag{19}$$

We call κ_c as a complex effective reflection coefficient (complex ERC). This quantity is defined without considering the liquid in contact with the LS. Therefore, κ_c is characteristic to the LS. Reflection rate in the substrate is $R = |r|^2$. We get transmission rate as $T = 1 - R$, because of the conservation of energy. From the expression of r in Eq. (18), we can recognize that κ_c is a keystone to master the reflection and the transmission of acoustic waves in the LS.

When materials are lossless for the acoustic energy both in the LS and in the liquid, and the reflected waves from the bottom of liquid bath are neglectable; we have to require a condition that κ is real as is defined in Eq. (10). (cf. appendix B.2.) In general cases, κ_c is complex number. To vanish r, we need that the real quantity κ should coincide with κ_c. For this coincidence, a condition of $\mathrm{Im}[\kappa_c] = 0$ is also required and it gives the following relation

$$\frac{\cosh N\zeta}{s(\zeta)} = \tan\beta\ g(\alpha,\beta). \tag{20}$$

This equality defines the relation between N and ω. (Dependency of ω is in the quantities α, β, and ζ.) Denoting the values of α and β with a subscript e on this condition, we express the real value of κ_c by κ_e and its expression is as follows

$$\kappa_e = \frac{-\frac{1}{2}(Z_A/Z_B - Z_B/Z_A)\ \sin\alpha_e\ \cos\beta_e}{g(\alpha_e,\beta_e)}. \tag{21}$$

We call κ_e an ERC without using 'complex' or 'real.' The quantity κ_e does not depend on N. For the condition $\mathrm{Im}[\kappa_c]=0$, we also obtain $\kappa_c = 0$ at frequencies which is derived from $s(\zeta)\ \sin\alpha = 0$. However, this case is the same as an usual impedance mismatch theory using the bulk acoustic impedance Z_B. Therefore, we do not discuss this case in detail. [When N=1 and $\mathrm{Im}[\kappa_c]$=0, the condition in Eq. (20) is reduced to Eq. (64), and κ_e in Eq. (21) is reduced to Eq. (65).] When the condition of $\mathrm{Im}[\kappa_c]$=0 is satisfied, we get $R = K^2$ where

$$K = \frac{\kappa - \kappa_e}{1 - \kappa\ \kappa_e}. \tag{22}$$

(This) quantity K corresponds to reflection coefficient without the phase factor. When a condition of $\kappa = \kappa_e$ is satisfied simultaneously with the condition of $\mathrm{Im}[\kappa_c]$=0, we get $R = 0$. Therefore, the transmission rate T is enhanced and becomes its maximum value, i.e., $T = 1$. For this enhancement, we also require that $|\kappa_e| < 1$, because κ has to satisfy an inequality $|\kappa| < 1$.

We define the EAI like

$$Z_e = Z_B\frac{1-\kappa_e}{1+\kappa_e} = Z_B\frac{(Z_A/Z_B)\tan\alpha_e + \tan\beta_e}{(Z_B/Z_A)\tan\alpha_e + \tan\beta_e}, \tag{23}$$

then the expression of K is reduced as

$$K = \frac{1 - Z_L/Z_e}{1 + Z_L/Z_e}. \tag{24}$$

When we compare this expression of K with the transmission coefficient κ in Eq. (10), we notice that the impedance Z_B of the solid B is substituted by Z_e. This feature shows a validity of the EAI which plays a role of characteristic impedance of the LS. Further, it is interesting that the number N is not explicitly included in the expression of Eq. (23). This means that the unit bilayer has the same expression of Z_e as well as that of a semi-infinite LS on the condition $\mathrm{Im}[\kappa_c]$=0.

3.2 Numerical examples

We show numerical examples for a system with LS consistent of Cu and Ag. The solid A is Cu and the solid B is Ag. The substrate of the system is Ag. Thicknesses of the metals are the same, i.e., d_A=0.50 mm and d_B=0.50 mm. Parameters of the metals are assumed as follows: For Cu, density is 8.93 g cm^{-3}, sound velocity is 5.01 km s^{-1}, then the acoustic impedance Z_A becomes 44.7×10^6 kg m^{-2} s^{-1}. For Ag, density is 10.5 g cm^{-3}, sound velocity is 3.65 km s^{-1}, then the acoustic impedance Z_B becomes 38.3×10^6 kg m^{-2} s^{-1}. In Fig. 2, the complex ERCs, κ_c's, are shown as a spiral for the layered structure Ag(CuAg)$_N$, where the total number of bilayers is set as N=11. The figure has three axes which are for frequencies, for real parts of κ_c's, and for imaginary parts of κ_c's. At many discrete frequencies, the imaginary parts of κ_c's vanish. At these points, the condition given in Eq. (20) is satisfied and it gives the ERCs or κ_e's. Two points labeled '1' and '2' are these instances to be discussed, and their frequencies are f_1=2.085

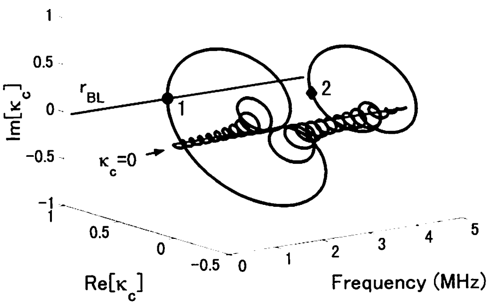

Figure 2: **The complex effective reflection coefficients for a layered structure of $Ag(CuAg)_{11}$. The effective acoustic impedances are defined at discrete frequencies on a condition that $Im[\kappa_c]=0$. Frequencies f_1=2.085 MHz and f_2=4.295 MHz are examples which satisfy the condition as marked in the figure. On the condition of $s(\zeta)\ \sin\alpha=0$, the spiral line of κ_c passes on the straight line labeled '$\kappa_c = 0$.'**

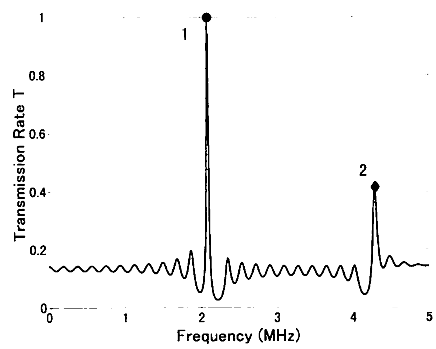

Figure 3: **The frequency dependency of transmission rates of acoustic waves through the interface $Ag(CuAg)_{11}$-water.**

MHz and f_2=4.295 MHz, respectively. The values of κ_e's at these two frequencies are κ_{e1}=0.926 and κ_{e2}=0.550; and the EAIs are Z_{e1}=1.48 and Z_{e2}=11.1 in unit of 10^6 kg m^{-2} s^{-1}, respectively. These values are smaller than those of Z_B and Z_A. Their ratios to Z_B are $Z_{e1}/Z_B \simeq 1/26$ and $Z_{e2}/Z_B \simeq 1/3.4$.

When the layered structure $Ag(CuAg)_{11}$ is in contact with water, we have transmission rate enhancement as in Fig. 3. Parameters of the water are 1.00 g cm^{-1} for density, and 1.48 km s^{-1} for sound velocity. The acoustic impedance is then 1.48×10^6 kg m^{-2} s^{-1}. This value coincide with Z_{e1}. Therefore, we find a maximum peak at the frequency f_1=2.085 MHz as labeled '1' in Fig. 3. At frequency f_2=4.295 MHz, Z_{e2} has a mismatch with the impedance of water, so that the transmission peak is not enhanced extremely and its peak value is 0.42.

When we substitute a material X for the water in contact with the structure $Ag(CuAg)_{11}$ at frequency f_1=2.085 MHz, the transmission rate changes as is shown in Fig. 4. The material X is expressed by the acoustic impedance Z_L. This curve is given by $T = 1 - K^2$ and Eq. (24), and is plotted against impedance ratios Z_L/Z_B on a log scale. On a point labeled '1,' the material X gives the same reflection coefficient κ_{e1} in $Ag(CuAg)_{11}$, and the transmission rate becomes its maximum value. For the sake of comparison, a curve for the interface B-X is displayed, where the bulk solid B is Ag.

At the frequency f_2=4.295 MHz, we plot a similar curve in Fig. 5. On a point labeled '2,' the system is the same as that of the point '2' in Fig. 3. On a point label '3' in Fig. 5, the transmission rate has its maximum value. This means that the material X gives the reflection coefficient κ_{e2}=0.550 in $Ag(CuAg)_{11}$, or X has the same impedance as Z_{e2} of $Ag(CuAg)_{11}$.

When the material X is fixed to have impedance Z_{e2}, the transmission rate has a frequency dependency as in Fig. 6. At frequency f_2=4.295 MHz, the transmission rate has the maximum value on a point labeled '3'. However, we get a lower peak at frequency f_1=2.085 MHz on a point labeled '4'. This lower peak originates from the large mismatch between the impedance of the material X and Z_{e1} of $Ag(CuAg)_{11}$. This is in contrast with the situation corresponding to the point labeled '1' in Fig. 3.

To see the N dependency, we plot the transmission rate for $Ag(CuAg)_N$-water at the frequency f_1=2.085 MHz in Fig. 7. One maximum peak is at N=11 which means that the system is on the condition $Im[\kappa_c]$=0 [or Eq. (20) is satisfied]. We also plot the transmission rate for $Ag(CuAg)_N$-X at the frequency f_2=4.295 MHz in Fig. 8 when the acoustic impedance of X is fixed as the same value as Z_{e2}=11.1 $\times10^6$ kg m^{-2} s^{-1}. Several peaks appear, which also mean that systems are on the

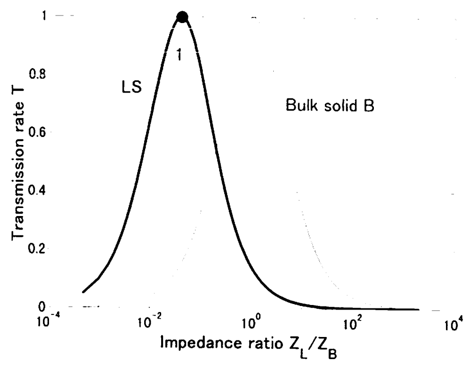

Figure 4: **The transmission rates of acoustic waves through Ag(CuAg)$_{11}$-X at frequency f_1=2.085 MHz. The material X is expressed by acoustic impedance Z_L. The impedance ratio to Z_B is on a log scale. For the sake of comparison, a curve for the interface Ag-X is displayed. (The solid B is Ag.)**

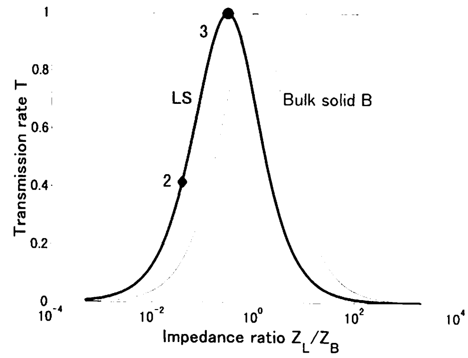

Figure 5: **The transmission rates of acoustic waves through Ag(CuAg)$_{11}$-X at frequency f_2=4.295 MHz. The material X is expressed by acoustic impedance Z_L. Impedance ratio with Z_B is on a log scale. For the sake of comparison, a curve for the interface Ag-X is also displayed. (The solid B is Ag.)**

conditions that Eq. (20) is satisfied. In these cases, the maximum peaks appears at N=11, 52, 93 with a period of 41. This dependency stems from Eq. (20). The values of ζ's are $\zeta_1 = i\pi - 0.146$ and $\zeta_2 = i0.0765$ at the frequencies f_1 and f_2, respectively. The condition in Eq. (20) is modified as

$$\tanh N\zeta = \frac{\sinh\zeta}{\tan\beta\ g(\alpha,\beta)}.$$

When we fix the frequency, then ζ, α, and β are also fixed. Therefore, the condition Eq. (20) with ζ_1 does not have periodicity at the frequency f_1. The condition with ζ_2 has a periodicity $\pi/|\zeta_2|$=41 at the frequency f_2. On the point labeled '1' in Fig. 7 and the point labeled '3' in Fig. 8, both numbers of bilayers are the same as 11 which makes the transmission rates be the maximum value.

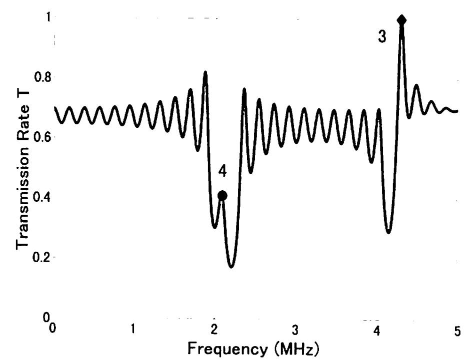

Figure 6: **The frequency dependency of transmission rates of acoustic waves through the interface Ag(CuAg)$_{11}$-X. The acoustic impedance of X is fixed as the same as Z_{e2}=11.1 $\times 10^6$ kg m^{-2} s^{-1}.**

4 Surface vibrations

4.1 Complex impedance in solid

We define the complex impedance in the solid as $Z(x) = -\sigma(x)/\dot{u}(x)$, where $\dot{u}(x) = -i\omega u(x)$; $u(x)$ and $\sigma(x)$ are the displacement and the stress defined by Eqs. (50) and (51), respectively. With the same method to define the complex impedance in the liquid (appendix B.2), we derive the following

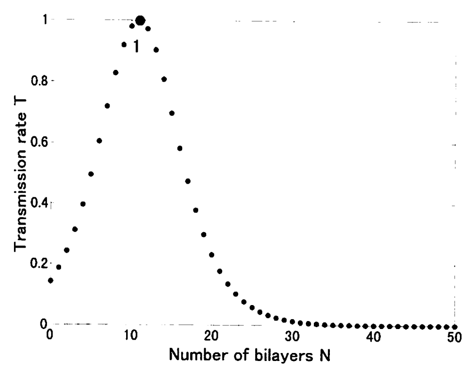

Figure 7: **The dependency on the total number of bilayers N in transmission rates of acoustic waves through the interface Ag(CuAg)$_N$-water.**

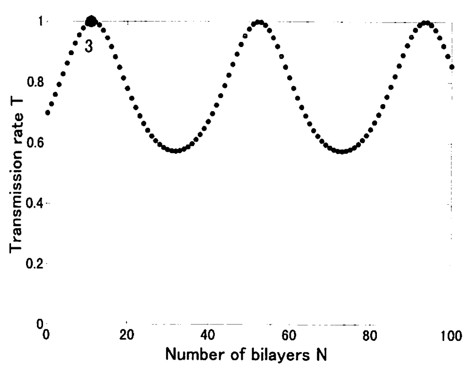

Figure 8: **The dependency on N in transmission rates of acoustic waves through the interface Ag(CuAg)$_N$-X. The acoustic impedance of X is fixed as Z_{e2}=11.1 $\times 10^6$ kg m^{-2} s^{-1}.**

two expressions

$$Z(0) = Z_S \frac{Z(\ell) - iZ_S \tan k_S\ell}{Z_S - iZ(\ell)\tan k_S\ell}, \qquad (25)$$

$$Z(x) = Z_S \frac{1 - \kappa\, e^{-i2k_Sx}}{1 + \kappa\, e^{-i2k_Sx}}, \qquad (26)$$

where Z_S is the acoustic impedance of solid, and k_S is the wave number of acoustic waves in the solid. The first expression, Eq. (25), is the relation of impedances at points separated by a distance ℓ. In the second expression, Eq. (26), the quantity κ is the ratio of amplitudes and its value is complex in general cases.

When we apply the above expressions to the solid B at the LS surface in contact with the liquid, the complex impedance at the interface ($x = 0$) becomes $Z(0) = Z_B\ (1-\kappa)/(1+\kappa)$. Further, on the conditions of Im[κ_c]=0 and $\kappa = \kappa_e$, $Z(0)$ becomes

$$Z(0) = Z_B \frac{1-\kappa_e}{1+\kappa_e} = Z_e. \qquad (27)$$

Therefore, the EAI is the impedance of LS surface on the conditions of transmission enhancement for acoustic waves transmitted from the LS substrate into liquid.

4.2 Energy flux and surface vibrations

The time averaged energy flux in bulk solid S is

$$J_S(x) = -\frac{1}{2}\,\mathrm{Re}[\sigma(x)^*\,\dot{u}(x)] = \frac{1}{2}\,|\sigma(x)|\,|\dot{u}(x)|\cos\theta(x), \qquad (28)$$

where $\theta(x)$ is the phase difference between $-\sigma(x)$ and $\dot{u}(x)$. This expression can be alternatively expressed with the complex impedance defined in the section 4.1 as follows

$$J_S(x) = \frac{1}{2}\frac{|\sigma(x)|^2}{|Z(x)|}\cos\theta(x) = \frac{1}{2}\,|Z(x)|\,|\dot{u}(x)|^2\cos\theta(x), \qquad (29)$$

where the phase is also expressed as $\theta(x) = \arg[Z(x)]$.

We next discuss energy fluxes in the LS. The flux in the substrate is expressed like

$$J_0 = \frac{1}{2}\,\omega^2\,Z_B\,|a_0|^2\,(1-|r|^2). \qquad (30)$$

The flux in the solid B of the LS surface is

$$J_N = \frac{1}{2}\,\omega^2\,Z_B\,|a_N|^2\,(1-|\kappa|^2). \qquad (31)$$

We can put $J_0 = J_N$ without any conditions because the system is lossless. Therefore, on the condition Im[κ_c]=0, the ratio of amplitude a_N in the LS surface solid to a_0 in the LS substrate becomes

$$|t|^2 = \left|\frac{a_N}{a_0}\right|^2 = \frac{(1-\kappa_e)(1+\kappa_e)}{(1-\kappa\,\kappa_e)^2}, \qquad (32)$$

where we use $|r| = K$ and Eq. (22). From this equality, we obtain

$$|t|^2 \to Z_B/Z_e \quad (\kappa \to 1). \qquad (33)$$

This means that the amplitude at the complete free surface is limited by a finite value. When Im[κ_c] $\neq 0$, then $|t|^2 \to (1-|\kappa_c|^2)/|1-\kappa_c|^2$ as $\kappa \to 1$. [We can also derive that $|t|^2 \to (1-|\kappa_c|^2)/|1+\kappa_c|^2$ as $\kappa \to -1$ for fixed surface, i.e., $Z(0) = \infty$.]

At the complete free surface, the impedance is $Z(0) = 0$. Further, in a case that $Z(0) = 0$ or $Z(0)$ is pure imaginary, the energy flux at every point in the LS vanishes. This feature is easily derived from Eqs. (25) and (29).

5 Energy incidence from liquid

We discuss the energy flow when the acoustic energy is incident from the liquid into the LS. In this case, the amplitude a_0 vanishes. From Eq. (18), this is equivalent to $|1/r| = 0$ or $\kappa = 1/\kappa_c^*$. We consider that no reflections exist in the LS substrate. This means that κ is always equal to $1/\kappa_c^*$. When κ_c^* is complex, the waves are propagating to directions both positive and negative with regard to the coordinate x in the solid B of the LS surface. We can modify Eq. (5) as follows

$$1/\xi_L = \frac{r_{BL} - \kappa_c^*}{1 - r_{BL}\,\kappa_c^*}, \tag{34}$$

where we substitute the solid B for the solid S, and $r_{BL} = (1 - Z_L/Z_B)/(1 + Z_L/Z_B)$ is the reflection coefficient at the interface ($x = 0$) between the solid B and the liquid L without any reflections in it. Using this expression, we obtain the energy flux in the liquid as follows

$$\begin{aligned} J_L^{inv} &= -\frac{1}{2}\,k_L^2\,Z_L\,|b_L|^2\,(1 - |1/\xi_L|^2) \\ &= -\frac{1}{2}\,k_L^2\,Z_L\,|b_L|^2\,\frac{(1 - r_{BL}^2)(1 - |\kappa_c|^2)}{|1 - r_{BL}\,\kappa_c|^2} \end{aligned} \tag{35}$$

where b_L is the velocity potential amplitude, and k_L the wave number in the liquid. The quantity $1/\xi_L$ corresponds to the amplitude reflection coefficient when the acoustic wave is incident from the liquid and reflected at the LS surface back into the liquid. We can accept that $r_{BL} < 1$ and $|1/\xi_L| < 1$. Therefore, we obtain $J_L^{inv} < 0$, i.e., the energy flows to the negative direction. From Eq. (35), we obtain the transmission rate from the liquid to the LS substrate as follows

$$T^{inv} = \frac{(1 - r_{BL}^2)(1 - |\kappa_c|^2)}{|1 - r_{BL}\,\kappa_c|^2}, \tag{36}$$

because the energy flux J_0^{inv} in the LS substrate is equal to J_L^{inv} and the factor $-\frac{1}{2}\,k_L^2\,Z_L\,|b_L|^2$ is the incident energy from the liquid. We can express as $T^{inv} = J_L^{inv}/(-\frac{1}{2}\,k_L^2\,Z_L\,|b_L|^2)$. The expression in Eq. (36) is reduced to exactly the same as that of the transmission rate T from the LS substrate to the liquid as discussed in the section 3.1. (In this case, r_{BL} is always equal to κ.) Therefore, the transmission from the liquid is also enhanced at the same frequencies for the transmission from the LS surface as shown in Fig. 3 for the system discussed in the section 3.2.

The surface vibrational magnitude is expressed as

$$\left|\frac{b_N}{b_0}\right|^2 = \frac{1}{1 - |\kappa_c|^2}, \tag{37}$$

where b_N and b_0 are the amplitudes in the LS surface and in the LS substrate, respectively. The amplitude at the LS surface becomes greater as $|\kappa_c|$ increases. If the enhancement conditions, $\mathrm{Im}\,[\kappa_c]=0$ and $\kappa = 1/\kappa_e$, are satisfied, the displacement of the LS surface is expressed as $u(0) = b_N(1 + \kappa_e)$, and the displacement in the substrate is b_0 (n.b. $a_0 = 0$). Because we are now assuming that the LS surface and the LS substrate are of the same solid B, the ratio of energy density deposited in the LS surface solid to that in the substrate becomes as follows

$$\left|\frac{u(0)}{b_0}\right|^2 = \left|\frac{b_N}{b_0}\right|^2 (1 + \kappa_e)^2 = Z_B/Z_e. \tag{38}$$

This means that the LS surface has higher energy density than that in the substrate on the enhancement conditions. Therefore, the gradient of energy density has large magnitude, and the thermal conductance is low at frequencies for the transmission enhancement.

From Eq. (26), the impedance of the LS surface becomes

$$Z(0) = -Z_B\frac{1 - \kappa_c^*}{1 + \kappa_c^*}. \tag{39}$$

Because $Z(0) = -\sigma(0)/\dot{u}(0)$, we obtain the stress on the LS surface as follows

$$|\sigma(0)| = \omega\;|u(0)|\;Z_B\;\left|\frac{1 - \kappa_c}{1 + \kappa_c}\right|, \tag{40}$$

where κ_c is the complex ERC and $\kappa_c = 1/\kappa^*$ (or $\kappa = 1/\kappa_c^*$) is related to the reflection coefficient $1/\xi_L$ as is expressed in Eq. (34). For the energy incidence from the liquid of the system discussed in the section 3.2, the ratios of surface impedances to the acoustic impedance of the LS surface solid B, $Z(0)/Z_B$, is shown in Fig. 9. At the frequency f_1=2.085 MHz labeled '1' in the figure, $|Z(0)|$ has the minimum value, and the ratio $Z(0)/Z_B$ is equal to $-Z_{e1}/Z_B = -1/26$. At the

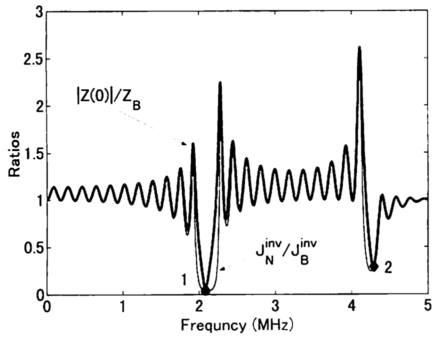

Figure 9: **The magnitude of impedance ratio $|Z(0)|/Z_B$ is plotted with thick line against frequencies. The energy is incident from the water of the system $(CuAg)_{11}$-water. At the frequency f_1=2.085 MHz labeled '1,' $|Z(0)|$ has the minimum value, and the ratio $Z(0)/Z_B$ is equal to $-Z_{e1}/Z_B = -1/26$. At the frequency f_2=4.295 MHz, the ratio $Z(0)/Z_B$ is equal to $-Z_{e2}/Z_B = -1/3.4$ as shown labeled '2.' The ratio of J_N^{inv}/J_B^{inv} is also plotted with thin line, which is almost the same with the line of $|Z(0)|/Z_B$ except near the points labeled '1' and '2.'**

frequency $f_2 = 4.295$ MHz, the ratio $Z(0)/Z_B$ is equal to $-Z_{e2}/Z_B = -1/3.4$ as shown labeled '2.' These values of $Z(0)$ are not affected by the liquid characteristics.

6 Discussion

The transmission rate is expressed as $T = (1-\kappa^2)|t|^2$ with assuming acoustic waves incident from the LS substrate into liquid [1]. On the condition that the transmission rate T has peak values, the amplitude reflection coefficient κ satisfies the following quadratic equation:

$$\kappa^2 - \frac{|1/\kappa_c|^2+1}{\mathrm{Re}[1/\kappa_c]}\kappa + 1 = 0. \qquad (41)$$

When $\mathrm{Im}[\kappa_c]$=0, the solutions are $\kappa = \kappa_e$ and $\kappa = 1/\kappa_e$. (A similar equation has been derived previously for a different LS system. [5]) In a case of $\kappa = \kappa_e$, the transmission enhancement is established from the LS substrate into the liquid. This case is discussed in the sections 3 and 4. In the section 5, the other case of $\kappa = 1/\kappa_e$ is mainly discussed. This case corresponds to the energy incidence from the liquid into the LS.

6.1 Enhancement conditions

In the case of energy incidence from the LS substrate, the condition that the transmission rate becomes unity is equivalent to $r = 0$. This condition is expressed as $\kappa = \kappa_c$. However, we should keep κ to be real in the case that no reflections exist in the liquid. Therefore, this condition also requires another condition $\mathrm{Im}[\kappa_c]$=0. Explicitly,

$$\frac{\cosh N\zeta}{s(\zeta)} = \tan\beta\, g(\alpha,\beta). \qquad (42)$$

[This is the same formula in Eq. (20).] If the transmission rate becomes unity, the equality $\mathrm{Im}[\kappa_c] = 0$ must always be satisfied. There are many cases that r vanishes when we change the acoustic wave frequency and the liquid acoustic impedance Z_L (or the amplitude reflection coefficient κ). However, the condition $\mathrm{Im}[\kappa_c] = 0$ is not affected by Z_L. Obviously, the resonant transmission with the surface vibrational mode [1] is one of them. The condition $\mathrm{Im}[\kappa_c] = 0$ expresses the relation between the number of bilayers in the LS, the frequency for the transmission enhancement, and the acoustic impedances of solids in the LS. On the condition, $\mathrm{Im}[\kappa_c] = 0$ satisfied, the effective impedance is $Z_e = Z_B(1-\kappa_e)/(1+\kappa_e)$ and the equality $\kappa = \kappa_e$ is equivalent to $Z_L = Z_e$. The real value of κ_c and Z_c are given as follows

$$\kappa_e = \frac{-\frac{1}{2}(Z_A/Z_B - Z_B/Z_A)\sin\alpha\cos\beta}{g(\alpha,\beta)}, \qquad (43)$$

$$Z_e = Z_B\frac{(Z_A/Z_B)\tan\alpha + \tan\beta}{(Z_B/Z_A)\tan\alpha + \tan\beta}. \qquad (44)$$

[We show the same formulae of Eqs. (21) and (23) again.]

In the case of the resonant transmission with the surface vibrational mode, the quantity ζ has real part. [11] The real part of this ζ causes the increase of the displacement amplitudes from the substrate to the interface of the finite LS in contact with the liquid. If we fix the structure of the unit bilayer in the LS, the right-hand side of Eq. (42) depends only on the acoustic wave frequency. Therefore, the variation of the number N of bilayers in the finite LS changes the *resonant* frequency, as well as ζ and Z_e. When we change the number N from the value for the resonant transmission, the reflection rate increases and we can expressed it as

$$R = |r|^2 = \left|\frac{1 - Z_L/Z_e}{1 + Z_L/Z_e}\right|^2 \qquad (45)$$

with making use of Eq. (24). This is a reason for the transmission rate, $T = 1 - R$, to have the N-dependence as is discussed in the section 3.2 with fixing the *acoustic wave* frequency as shown in Figs. 7 and 8. From the above discussion, we can conclude that the displacement ratio does not increase exponentially in the finite LS, even if the ζ has an real part. [12]

If $\tan\alpha = 0$ in Eq. (44) simultaneously with Eq. (42) satisfied, then $Z_e = Z_B$ and the LS looks like a bulk solid B. Similarly, if $\tan\beta = 0$ in Eq. (44) simultaneously with Eq. (42) satisfied, then $Z_A = \sqrt{Z_e Z_B}$ and the layer A plays a role like a quarter-wave plate.

The quantities $e^{+\zeta}$ and $e^{-\zeta}$ are the eigenvalues of the transfer matrix G defined for the unit bilayer, and "$(Z_A/Z_B)\tan\alpha + \tan\beta = 0$" is the dispersion relation given by the condition that the stress vanishes at the free surface for semi-infinite layered media [5, 11] (not for finite-size layered media). If the relation "$(Z_A/Z_B)\tan\alpha + \tan\beta = 0$" is satisfied in Eq. (44) simultaneously with Eq. (42) satisfied, then the effective acoustic impedance becomes as $Z_e = 0$. This means that the displacement appears on the free surface even if the stress vanishes. However, $|t|^2 \to \infty$ for the free surface ($\kappa \to 1$) from Eq. (33). The loss in the system must be introduced for realistic considerations.

We discuss some conditions for $\tan\alpha$ and $\tan\beta$ satisfied with Im[κ_c]=0. However, they are not easy to be satisfied, because they are closely related with the unit bilayer. If we change the structure of unit bilayer, then the enhancement conditions, Im[κ_c]=0 and $\kappa = \kappa_e$, are also affected.

6.2 Energy flux near the LS surface

We have also discussed the case that the energy is incident from the liquid. In this case, the quantity κ must be always equal to $1/\kappa_c^*$. Further, when $\kappa_c^* = r_{BL}$, we have Im[κ_c]=0 and $1/\xi_L = 0$. [This also means that $\kappa = 1/r_{BL}$.] On these condition, the whole energy from the liquid penetrates into the LS surface. Therefore, transmission is also enhanced from the liquid into the LS substrate, that is the opposite direction discussed in the section 3. The condition Im[κ_c]=0 means that the enhancement condition is $\kappa = 1/\kappa_e$. The impedance at the LS surface becomes $Z(0) = -Z_e$ from Eq. (26). From Eq. (29), we obtain the energy flux at the LS surface as follows

$$J_N^{inv} = -\frac{1}{2} Z_e |\dot{u}(0)|^2. \tag{46}$$

[This is equivalent to $J_N^{inv} = \frac{1}{2} Z_B \omega^2 |a_N|^2 (1 - 1/\kappa_e^2) = -\frac{1}{2} Z_B \omega^2 |b_N|^2 (1-\kappa_e^2)$.] The energy flux in the bulk solid B is

$$J_B^{inv} = -\frac{1}{2} Z_B |\dot{u}(0)|^2. \tag{47}$$

(In a case of the LS, this expression is for the energy flux at frequencies given by the condition $\kappa_c = 0$.) When the deposit energy density, $\frac{1}{2}\rho_B |\dot{u}(0)|^2 = \frac{1}{2}\rho_B \omega^2 |u(0)|^2$, is the same in both systems at the coordinate $x = 0$, then $|\dot{u}(0)|^2$ has the same value in Eqs. (46) and (47). This assumption expresses that temperature is the same at $x = 0$ in the both systems. Therefore, we have a relation $J_N^{inv}/J_B^{inv} = Z_e/Z_B$.

At general frequencies, the ratio above becomes

$$\frac{J_N^{inv}}{J_B^{inv}} = \frac{|Z(0)\cos\theta(0)|}{Z_B} \tag{48}$$

from Eq. (29), which is plotted also in Fig. 9 for the same system discussed in the section 3.2. If frequency channels are open for the thermal conduction by lower EAIs or by higher $|\kappa_c|$, then we can expect that the thermal conductance near the LS surfaces reduced than that of the bulk solid B by a factor of 10 or more. The lower EAI implies that the group velocity of the acoustic waves becomes slower (as in a delay line of spatial harmonic wave tube in a field of microwave engineering). When we cannot set $\kappa_c = 1$ in Eq. (40), we cannot remove the stress on the surface of the finite LS. This means that $Z(0) \neq 0$ and the energy flux cannot remove from the system as is discussed in the section 4.2. [From Eq. (39), the condition that $Z(0)$ becomes zero or pure imaginary is derived as $|\kappa_c| = 1$.]

If we could assume that the liquid plays like an electromagnetic field, the laser light of the pump and probe experiments [12] should give a finite stress on the LS surface. If the LS surface is faced to vacuum, we can assume the stress may be weak, i.e. $\kappa_c \sim 1$, and the surface vibrational magnitude may increase as expressed in Eq. (37). In the case of Im[κ_c]=0, the above discussion implies that higher values of $|\kappa_e|$ or lower values of Z_e are realistic, and the stress in Eq. (40) becomes $|\sigma(0)| = \omega |u(0)| Z_e$. Therefore, the energy flux at the surface is reduced than that in a bulk solid. This discussion gives us a feasibility to understand low thermal conductance in LS's. [6, 7] However, we need to consider the state density of the LS system [8] for the sake of further studies.

7 Conclusion

With making use of liquid as a working material, we derive the EAI which is characteristic to the finite-size LS and it is not affected by the materials in contact with the LS surface. The EAI gives concise expressions of transmission rates or reflection rates, and their frequency dependencies are exactly the same for acoustic energy flows from the LS substrate into liquid and from liquid into the LS. In the case of transmission from the LS substrate into liquid, the transmission rate becomes unity on the conditions of Im[κ_c]=0 and $\kappa = \kappa_e$ (or $Z(0) = Z_e = Z_L$). In another case of transmission from liquid into the LS, the conditions are Im[κ_c]=0 and $\kappa = 1/\kappa_e$ (or $Z(0) = -Z_e = -Z_L$). In both these cases, the transmission is enhanced at frequencies given by the condition Im[κ_c]=0. As a result, the EAI is defined at discrete frequencies, and the expression of EAI does not depend explicitly on the total number of bilayers in the LS.

In the case that the transmission rate T becomes unity for acoustic energy flows from the LS substrate into liquid, the surface vibrational amplitude is limited by the ratio of Z_B/Z_e, even if the LS surface is completely free. Its magnitude grows larger as κ_e approaches near the value of unity.

In the case that $T^{inv} = 1$ for the transmission from liquid into the LS, the ratio of deposit energy density in the LS surface to that in the LS substrate becomes Z_B/Z_e. If Z_e is less than Z_B at the enhancement frequencies, the ratio becomes larger and the thermal conductance becomes lower than that of bulk solid or the LS with $\kappa_c = 0$. The LS surface cannot be completely free. Therefore, the stress on the LS surface is nonzero in stationary states as discussed in this paper.

Finally, the conclusion of the present study denies the possibility discussed in the previous paper [1] that the thermal resistance decreases between metals and liquids like helium at low temperatures when we use finite-size layered metals instead of bulk metals. It seems like a contradiction, because the transmission rate becomes unity on the condition of the resonant transmission (or the transmission enhancement with $\kappa_e \simeq 1$). However, the resonant condition gives low acoustic impedance at the LS surface which causes low thermal conductance.

This work was supported in part by a Grant-in-Aid for Scientific Research from the Ministry of Education, Science and Culture of Japan (Grant No. 11650062).

A Solid system

A.1 Matrix notations for solid

First, we discuss displacements and stresses caused by the acoustic field in solids. [13, 14] Transmissions and reflections of the acoustic waves are expressed with making use of matrix notations.

One dimensional equation of motion for acoustic waves in a solid S is reduced, with the assumption that the time dependency is $e^{-i\omega t}$, as follows

$$\frac{\partial^2 u(x)}{\partial x^2} + \frac{\rho_S\, \omega^2}{c_{11}}\, u(x) = 0, \tag{49}$$

where ω is the angular frequency of acoustic vibration, and x is the space coordinate; $u(x)$ is the longitudinal displacement at the coordinate x, ρ_S is the density, and c_{11} is the stiffness constant of solid S. A solution has a form like

$$u(x) = a_S\, e^{ik_S\, x} + b_S\, e^{-ik_S\, x}, \tag{50}$$

where $k_S = \omega\sqrt{\rho_S / c_{11}}$, a_S and b_S are displacement amplitudes for travelling waves directed to the positive direction and the negative direction with respect to the coordinate x. The stress at x is then expressed as

$$\sigma(x) = c_{11}\frac{\partial\, u(x)}{\partial x} = i\omega Z_S\, a_S\, e^{ik_S\, x} - i\omega Z_S\, b_S\, e^{-ik_S\, x}, \tag{51}$$

where Z_S is the acoustic impedance which is defined as $Z_S = \rho_S\, v_S$ and $v_S = \omega/k_S$ is the sound velocity in the solid.

If there is an interface of two solids at $x = 0$, then the continuity of displacement and the stress is expressed as $u(-0) = u(+0)$ and $\sigma(-0) = \sigma(+0)$. These relations are expressed more conveniently with using the following matrix notations.

$$\Psi_S(x) = \begin{bmatrix} \dot{u}(x) \\ -\sigma(x) \end{bmatrix} = M_S\ \Phi_S(x)\ \mathbf{W}_S, \tag{52}$$

where we define that the displacement velocity is $\dot{u}(x) = -i\omega\, u(x)$, and

$$M_S = -i\omega \begin{bmatrix} 1 & 1 \\ Z_S & -Z_S \end{bmatrix}, \quad (53)$$

$$\Phi_S(x) = \begin{bmatrix} e^{ik_S\, x} & 0 \\ 0 & e^{-ik_S\, x} \end{bmatrix}, \quad (54)$$

$$\mathbf{W}_S = \begin{bmatrix} a_S \\ b_S \end{bmatrix}. \quad (55)$$

Therefore, the condition of continuity is expressed by $\Psi_S(-0) = \Psi_S(+0)$.

A.2 Effective acoustic impedance of bulk solid

We now discuss the EAI of bulk solid. In a case of the semi-infinite solid S with a surface at $x = 0$ and with an acoustic wave source at $x = -d$, we express amplitude vector at the surface by $\mathbf{W}(0)$ whose components are a_S and b_S, and amplitude vector at the source by $\mathbf{W}(-d)$ whose components are $a_S e^{-ik_S d}$ and $b_S e^{ik_S d}$. Relation of these two vectors is expressed as $\mathbf{W}(0) = \Phi_S(d)\ \mathbf{W}(-d)$, where $\Phi_S(d)$ is given by Eq. (54). The matrix $\Phi_S(d)$ is a transfer matrix for the semi-infinite solid. Therefore, the complex ERC κ_c becomes zero. This means $Z_e = Z_S$ without any conditions. If the solid is in contact with a material with the impedance Z_S, transmission rate becomes unity. This is a trivial example, but shows that no contradictions are caused by the EAI.

A.3 Effective acoustic impedance of unit bilayer

To discuss a LS in the section 3, we consider a system consistent of solids A, B, and X. We denote a layered configuration as $BABX$, where the solids are connected in an order as noted. The left solids in $BABX$ are at coordinates less than those of the right solids. When we express an interface of two solids, we use '-' between them like BAB-X. Interfaces of the solids are assume to exit at $x = 0$ for BAB-X, at $x = -d_B$ for BA-BX, and at $x = -d_A - d_B$ for B-ABX. Making use of the matrices defined above, we get a relation between displacement amplitudes at $x = -d_A - d_B - 0$ in the solid B and those at $x = -0$ in the other solid B as $\mathbf{W}(-0) = G\ \mathbf{W}(-d_A - d_B - 0)$, where

$$G = \Phi_S^{(B)}(d_B)\ f^{(AB)}\ \Phi_S^{(A)}(d_A)\ f^{(BA)}, \quad (56)$$

$$f^{(AB)} = \frac{1}{2Z_B} \begin{bmatrix} Z_B + Z_A & Z_B - Z_A \\ Z_B - Z_A & Z_B + Z_A \end{bmatrix}, \quad (57)$$

$$f^{(BA)} = \frac{1}{2Z_A} \begin{bmatrix} Z_A + Z_B & Z_A - Z_B \\ Z_A - Z_B & Z_A + Z_B \end{bmatrix}, \quad (58)$$

$$\Phi_S^{(A)}(d_A) = \begin{bmatrix} e^{ik_A d_A} & 0 \\ 0 & e^{-ik_A d_A} \end{bmatrix}, \quad (59)$$

$$\Phi_S^{(B)}(d_B) = \begin{bmatrix} e^{ik_B d_B} & 0 \\ 0 & e^{-ik_B d_B} \end{bmatrix}. \quad (60)$$

In these expressions, d_A and d_B are thicknesses of a solid layer A and a solid layer B, respectively. The phase changes of amplitudes are expressed by $\Phi_S^{(A)}(d_A)$ and $\Phi_S^{(B)}(d_B)$ for the waves to travel in the layers. Wave numbers are denoted as k_A and k_B in the layer A and the layer B, respectively. The matrix G is a transfer matrix which defines the relation of displacement amplitudes separated just by one bilayer AB. Explicit expressions for the elements of G are given as follows

$$G_{11} = e^{i\beta}\ \{\cos\alpha + \frac{i}{2}(Z_A/Z_B + Z_B/Z_A)\sin\alpha\}, \quad (61)$$

$$G_{12} = e^{i\beta}\ \frac{i}{2}(Z_A/Z_B - Z_B/Z_A)\sin\alpha, \quad (62)$$

and $G_{22} = G_{11}^*$, $G_{21} = G_{12}^*$, where $\alpha = k_A d_A$, and $\beta = k_B d_B$. Therefore, we get the complex ERC ($\kappa_c = G_{21}/G_{11}$) as

$$\kappa_c = \frac{-i\ e^{-i2\beta}\frac{1}{2}(Z_A/Z_B - Z_B/Z_A)\sin\alpha}{\cos\alpha + \frac{i}{2}(Z_A/Z_B + Z_B/Z_A)\sin\alpha}. \quad (63)$$

The condition $\mathrm{Im}[\kappa_c] = 0$ is given as follows

$$\frac{1}{2}(Z_A/Z_B + Z_B/Z_A)\tan\alpha\tan 2\beta = 1. \quad (64)$$

This condition gives frequencies which make the transmission be enhanced. The ERC becomes

$$\begin{aligned} \kappa_e &= -\frac{1}{2}(Z_A/Z_B - Z_B/Z_A)\tan\alpha\sin 2\beta \\ &= \frac{-\frac{1}{2}(Z_A/Z_B - Z_B/Z_A)}{\frac{\tan\beta}{\tan\alpha} + \frac{1}{2}(Z_A/Z_B + Z_B/Z_A)}, \end{aligned} \quad (65)$$

and the EAI of the structure BAB is derived by $Z_e = Z_B(1 - \kappa_e)/(1 + \kappa_e)$. The explicit form becomes

$$Z_e = Z_B\frac{(Z_A/Z_B)\tan\alpha + \tan\beta}{(Z_B/Z_A)\tan\alpha + \tan\beta}. \quad (66)$$

This expression has the same form as that of the LS with N bilayers.

B Liquid or fluid

B.1 Matrix notations for liquid

To understand the transmission of acoustic waves from solid into liquid, we make up the matrix notations of liquid or fluid in this section. If we neglect the nonlinear term and the viscosity in the Navier-Stokes equation, a velocity potential $\phi(x)$ is defined with the equation of continuity, and satisfies the following equation

$$\frac{\partial^2\,\phi(x)}{\partial x^2}+\frac{\omega^2}{c_o^2}\,\phi(x)=0, \qquad (67)$$

where c_o is the sound velocity in one dimensional liquid L. [15] The time dependency is also assumed as $e^{-i\omega t}$ as the same way in the discussions of solid. This equation has a solution like

$$\phi(x)=a_L\,e^{ik_L\,x}+b_L\,e^{-ik_L\,x}, \qquad (68)$$

where $k_L=\omega/c_o$. With this solution, we can derive the acoustic field velocity $v=\partial\phi/\partial x$ and pressure deviation from its averaged value as $p'=-\rho_L\,\partial\phi/\partial t$, where ρ_L is averaged density. Their expressions are

$$v(x)=\frac{\partial\,\phi(x)}{\partial x}=ik_L\,a_L\,e^{ik_L\,x}-ik_L\,b_L\,e^{-ik_L\,x}, \qquad (69)$$

$$\begin{aligned}p'(x)&=i\omega\,\rho_L\,\phi(x)\\&=i\omega\,\rho_L\,a_L\,e^{ik_L\,x}+i\omega\,\rho_L\,b_L\,e^{-ik_L\,x}.\end{aligned} \qquad (70)$$

As the same way in the discussions of solid, we also define the following matrix notations

$$\Psi_L(x)=\begin{bmatrix}v(x)\\p'(x)\end{bmatrix}=M_L\,\Phi_L(x)\,\mathbf{W}_L, \qquad (71)$$

$$M_L=ik_L\begin{bmatrix}1&-1\\Z_L&Z_L\end{bmatrix}, \qquad (72)$$

$$\Phi_L(x)=\begin{bmatrix}e^{ik_L\,x}&0\\0&e^{-ik_L\,x}\end{bmatrix}, \qquad (73)$$

$$\mathbf{W}_L=\begin{bmatrix}a_L\\b_L\end{bmatrix}, \qquad (74)$$

where $Z_L=\rho_L\,c_o$ is acoustic impedance of the liquid. With these matrices, we can express the continuity of the acoustic field with that in solid.

B.2 Complex impedance of liquid

The acoustic impedance Z_L is a ratio $p'(x)/v(x)$ in a case that waves are travelling only to one direction, e.g., $b_L=0$. Even if there are waves travelling to the positive and negative directions simultaneously, we define the ratio $p'(x)/v(x)$ and call it complex impedance $Z(x)$. There are two expressions for $Z(x)$. One is given by expressions $\Psi_L(x)=M_L\;\Phi_L(x)\;\mathbf{W}_L$ and its modified form of $\mathbf{W}_L=[\Phi_L(x)]^{-1}\;[M_L]^{-1}\;\Psi_L(x)$. The coordinate x in these two expressions can be put independently, and $\mathbf{W}_L$ is allowed to be the same in both expressions. We put $x=0$ in the former and $x=\ell$ in the latter, and get $\Psi_L(0)=M_L\;[\Phi_L(\ell)]^{-1}[M_L]^{-1}\;\Psi_L(\ell)$. In explicit expressions, this equality becomes

$$\begin{bmatrix}v(0)\\p'(0)\end{bmatrix}=\begin{bmatrix}\cos k_L\ell&-i\frac{1}{Z_L}\sin k_L\ell\\-iZ_L\sin k_L\ell&\cos k_L\ell\end{bmatrix}\begin{bmatrix}v(\ell)\\p'(\ell)\end{bmatrix}. \qquad (75)$$

From this expression, we get a relation of the complex impedances $Z(0)=p'(0)/v(0)$ and $Z(\ell)=p'(\ell)/v(\ell)$ as follows

$$Z(0)=Z_L\frac{Z(\ell)-iZ_L\tan k_L\ell}{Z_L-iZ(\ell)\tan k_L\ell}. \qquad (76)$$

This is an expression that relates the complex impedances separated by a distance ℓ. The other expression is given by Eqs. (70) and (69) as

$$Z(x)=Z_L\frac{1+\xi_L\;e^{-i2k_Lx}}{1-\xi_L\;e^{-i2k_Lx}}, \qquad (77)$$

where $\xi_L=b_L/a_L$ and it means an amplitude reflection coefficient in the liquid. (We do not need to specify that ξ_L is defined at an interface nor at a surface.) These two expressions mean that the complex impedance $Z(x)$ is defined by standing waves in the liquids, because they have similar forms to the impedances of electrical transmission lines. However, Z_L in lossless liquid defined in this section corresponds to the reactive impedance in electricity. Differences of signs in Eqs. (76) and (77) with those of electrical transmission lines are caused by the definition of phase factor. It is $e^{-ikx+i\omega t}$ in electricity. When we cannot neglect reflected waves from the bottom of liquid bath (see Fig. 1), then $\xi_L\neq 0$ and we have to consider the complex impedance $Z(x)$. The reflection coefficient κ in Eq. (6) [Eq. (10)] must be substitute with

$$\kappa=\frac{1-Z(0)/Z_S}{1+Z(0)/Z_S}. \qquad (78)$$

The above reflection coefficient κ has complex values in general cases. If we introduce a viscosity in liquids [4] or we consider reflections in the liquid, the complex acoustic impedance $Z_c = Z_S(1-\kappa_c)/(1+\kappa_c)$ is realistic quantity in a solid for a discussion of acoustic wave transmissions through a solid-liquid interface.

References

[1] H. Kato, Phys. Rev. B **59**, 11 136 (1999): On this occasion, the author corrects an error of the equation (12) "$2\ \kappa^3 - 3\ \mathrm{Re}[c]\ \kappa^2 + (|c|^2 - 1)\ \kappa + \mathrm{Re}[c] = 0$" in the referred paper to "$\kappa^2 - \kappa\ (|c|^2+1)/\mathrm{Re}[c] + 1 = 0$." The quantity c is equivalent to $1/\kappa_c^*$ in text.

[2] S. Tamura, Phys. Rev. B **43**, 12 646 (1991).

[3] G. Tas, and H. J. Maris, Phys. Rev. B **55**, 1852 (1997).

[4] H. T. O'Neil, Phys. Rev. **75**, 928 (1949).

[5] Wei Chen, Yu Lu, Humphrey. J. Maris, and Gang Xiao, Phys. Rev. B **50**, 14 506 (1994).

[6] W. S. Capinski, M. Cardona, D. S. Katzer, H. J. Maris, K. Ploog, and T. Ruf, Physica B **263-264**, 530 (1999).

[7] W. S. Capinski, H.J. Maris, T. Ruf, M. Cardona, K. Ploog, and D. S. Katzer, Phys. Rev. B **59**, 8105 (1999).

[8] S. Tamura, Y. Tanaka, and H. J. Maris, Phys. Rev. B **60**, 2627 (1999).

[9] This equality is reduced as $M_A\ \Phi_A(0)\ \mathbf{W}_A = M_B\ \Phi_B(0)\ \mathbf{W}_B$. The matrices $\Phi_A(0)$ and $\Phi_B(0)$ becomes unit matrix. As a result, we obtain $\mathbf{W}_B = f^{(AB)}\ \mathbf{W}_A$, where $f^{(AB)} = [M_B]^{-1}\ M_A$.

[10] This equality is modified with the matrix notations as $\mathbf{W}_L = f^{(SL)}\ \mathbf{W}_S$, where $f^{(SL)} = [M_L]^{-1}\ M_S$.

[11] R. E. Camley, B. Djafari-Rouhani, L. Dobrzynski, and A. A. Maradudin, Phys. Rev. B **27**, 7318 (1983).

[12] H. T. Grahn, H. J. Maris, J. Tauc, and B. Abeles, Phys. Rev. B **38**, 6066 (1988).

[13] H. Kato, J. Acoust. Soc. Am. **101** (3), 1380 (1997).

[14] B. A. Auld, *"Acoustic Fields and Waves in Solids 2nd Ed.,"* Robert E. Krieger Publishing Co., Malabar, Florida (1990).

[15] L. D. Landau and E. M. Lifshitz, *"Fluid Mechanics 2nd Ed.,"* Pergamon Press, New York (1987).

(平成13年11月19日受理)