On the Semiranked Group (II)

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Synopsis

In this paper we will give a definition of the Direct Product of $(S)R$-groups and prove several propositions.

Throughout this paper we shall use the same terminology that is introduced in [1] and [2].

§6. The definition of the Direct Product of $(S)R$-groups. Let G_λ ($\lambda \in \Lambda$; Λ is finite) be R-spaces with indicater ω_0. And let $G = \prod_{\lambda\in\Lambda} G_\lambda$ be the direct product set of $\{G_\lambda;\ \lambda \in \Lambda\}$. Now, let $u_m^{(\lambda)}(x_\lambda)$ be an any neighborhood with rank α_λ of $x_\lambda \in G_\lambda$. Then, we define the neighborhood of $x = (x_\lambda)_{\lambda\in\Lambda} \in G$ as follows:

For $\forall x \in G, u_m(x) = (u_m^{(\lambda)}(x_\lambda))_{\lambda\in\Lambda}$ $(\subseteq G)$. Furthermore, let $\mathfrak{B}_n(G)$ and $\mathfrak{B}_n(G_\lambda)$ $0 \leqq \forall n < \omega_0)$, be two families of all neighborhoods in G and G_λ, respectively, such that

$$u_m(x) \in \mathfrak{B}_n(G) \quad (\text{for all } x \in G) \quad (n = \min\{\alpha_\lambda;\ \lambda \in \Lambda\})$$

$$\overset{\text{def.}}{\Longleftrightarrow} \quad u_m^{(\lambda)}(x_\lambda) \in \mathfrak{B}_n(G_\lambda) \quad (\text{for } \forall_\lambda \in \Lambda).$$

Then, G becomes an R-space. with indicater ω_0. (c. q. f. d.)

Definition 9. R-Direct Product space. For $\forall_\lambda \in \Lambda$, let G_λ be an R-space. Then the direct product set $G = (G_\lambda)_{\lambda\in\Lambda}$ is called the R-Direct Product space.

Thus, we get following proposition:

Proposition 11. The group operation in the Direct Product Group G is continuous in the R-Direct Product space G. Namely, G is an R (and SR)-group.

Proof. Let

$$x = (x_\lambda)_{\lambda\in\Lambda}, \qquad y = (y_\lambda)_{\lambda\in\Lambda}$$

be two any elements of G. And we put

$$xy^{-1} = z = (z_\lambda)_{\lambda\in\Lambda} \qquad \text{i. e.} \qquad z_\lambda = x_\lambda y_\lambda^{-1} \ (\lambda \in \Lambda).$$

Moreover, let

$$u_n(x) = (u_n^{(\lambda)}(x_\lambda))_{\lambda\in\Lambda}, \qquad v_n(y) = (v_n^{(\lambda)}(y_\lambda))_{\lambda\in\Lambda}$$

be any neighborhoods of x, y, respectively.

Since the group operation: $(x_\lambda, y_\lambda) \to x_\lambda y_\lambda^{-1}$ is continuous in each R-space G_λ ($\lambda \in \Lambda$), i. e., for any $\{u_n^{(\lambda)}(x_\lambda)\}$, $\{v_n^{(\lambda)}(y_\lambda)\}$ and all $\lambda \in \Lambda$, there exists a $\{w_n^{(\lambda)}(x_\lambda y_\lambda)\}$ such that

$$u_n^{(\lambda)}(x_\lambda)\, v_n^{(\lambda)}(y_\lambda)^{-1} \subseteq w_n^{(\lambda)}(x_\lambda\, y_\lambda^{-1}).$$

Therefore, for any $\{u_n(x)\}$, $\{v_n(y)\}$, there exists a $\{w_n(xy)\}$ such that

$$u_n(x)v_n(y)^{-1} \subseteq w_n(xy^{-1}). \qquad \text{(Q. E. D.)}$$

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Namely, we get next Proposition :

Proposition 12.

If, for each $\lambda \in \Lambda$, G_λ is a *SR*-group (or *R*-group), so is the Direct Product Group $G = \prod_{\lambda \in \Lambda} G_\lambda$.

Therefore,

Definition 10. If, for $\forall \lambda \in \Lambda$, G_λ is a *SR*-group (resp. *R*-group), then $\prod_{\lambda \in \Lambda} G_\lambda$ is called the *SR*-Direct Product Group (resp. *R*-Direct Product Group).

Furthermore, by the above arguments we get followings:

Proposition 13. Let $G = \prod_{\lambda \in \Lambda} G_\lambda$ be the Direct Product of *R*-spaces. Then the λ-th projection mapping

$$Pr_\lambda : G \to G_\lambda \ (\lambda \in \Lambda)$$

is *r*-continuous and

$$Pr_\lambda \ (u_n(x)) = u_n^{(\lambda)} \ (x_\lambda)$$

such that $u_n(x) = \prod_{\lambda \in \Lambda} u_n^{(\lambda)} (x_\lambda)$.

Proposition 14.

For each $\lambda \in \Lambda$, Pr_λ is a continuous homomorphism of $G = \prod_{\lambda \in \Lambda} G_\lambda$ onto the *SR*-group G_λ.

Proof. For $\forall \ x, y \in G$, $Pr_\lambda(xy) = x_\lambda y_\lambda = Pr_\lambda(x) \cdot Pr_\lambda(y)$.

Therefore, Pr_λ is a homomorphism. (Q. E. D.)

(To be continued.)

References

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